FINAL
MAT 141
12/22/00

Name	Sec.

ID number

1	2	3	4	5	6	total

THIS EXAM IS WORTH 80 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. YOU MAY USE THE BACKS OF PAGES FOR SCRATCH WORK.

1. **(2 pt each, 40 pts total)** Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

   (i) ☐ The function $f(x) = x^2 - x + 1$ is **(a)** never negative **(b)** always negative **(c)** always increasing **(d)** always decreasing **(e)** concave down **(f)** none of these.

   (ii) ☐ What is the maximum value of $f(x) = x - 2x^4$? **(a)** 3/16 **(b)** 3/8 **(c)** 1/8 **(d)** 1/4 **(e)** 1/2 **(f)** none of these.

   (iii) ☐ If a ball is dropped from 64 feet up, how fast is it going when it hits the ground (assume constant acceleration of $32ft/sec^2$)? **(a)** $16ft/sec$ **(b)** $32ft/sec$ **(c)** $48ft/sec$ **(d)** $56ft/sec$ **(e)** $64ft/sec$ **(f)** none of these.

   (iv) ☐ Find the slope of $y^2 + x^2 = y^4 - 2x$ at the point $(-2, 1)$ **(a)** 2 **(b)** 1 **(c)** 0 **(d)** -1 **(e)** -2 **(f)** none of these.

   (v) ☐ The definition of $f'(x)$ is **(a)** $\lim_{h\to 0} \frac{f(x+h)-f(h)}{h}$ **(b)** $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ **(c)** $\lim_{x\to 0} \frac{f(x+h)-f(x)}{h}$ **(d)** $\lim_{h\to x} \frac{f(x+h)-f(x)}{h}$ **(e)** $\lim_{h\to 0} \frac{f(x+h)-f(x)}{x}$ **(f)** none of these.

   (vi) ☐ Suppose $g(t) = (x^2 + \sin(x))^3$. Then $g'(t) =$ **(a)** $3(2x + \cos(x))^2$ **(b)** $3(x^2 - \cos(x))^2(2x + \sin(x))$ **(c)** $(x^2 + \sin(x))^3(2x + \cos(x))$ **(d)** $3(x^2 + \sin(x))^2(2x + \cos(x))$ **(e)** $3(x^2 + \sin(x))^2(2x - \cos(x))$ **(f)** none of these.

   (vii) ☐ The base of a solid is the region in the first quadrant between the line $y = x$ and the curve $y = 2\sqrt{x}$. The cross sections (perpendicular to the x-axis) of the solid are squares whose bases stretch from the line to the curve. The volume of the solid is **(a)** $\int_0^4 \frac{1}{2}(x - 2\sqrt{x})^2 dx$ **(b)** $\int_0^4 (x - 2\sqrt{x})^2 dx$ **(c)** $\int_0^2 (x - 2\sqrt{x})^2 dx$ **(d)** $\int_0^4 \frac{1}{2}(x - 2\sqrt{x}) dx$ **(e)** $\int_0^2 (x - 2\sqrt{x}) dx$ **(f)** none of these.

(viii) ☐ Suppose $\int_0^{f(x)} t^2 dt = x^2$. Then $f(x) =$ **(a)** x **(b)** $3^{1/3}x^{2/3}$ **(c)** x^2 **(d)** $2x^2$ **(e)** $x^{-2/3}$ **(f)** none of these.

(ix) ☐ Suppose $G(x) = \int_0^x \cos(t^2)dt$. Then $G''(x) =$ **(a)** $\sin^2(x)$ **(b)** $\cos(x^2)2x$ **(c)** $-2x\sin(x^2)$ **(d)** $\cos(x)2x$ **(e)** $-\sin(x)2x$ **(f)** none of these.

(x) ☐ Find the equation of the curve in the xy-plane that passes through the point $(1, 3)$ if its slope at x is always $3x^2 + 2$. **(a)** $y = x^3 + 2x$ **(b)** $y = 5$ **(c)** $y = 3x^2 - 2$ **(d)** $y = x^3 + 2x - 4$ **(e)** $x = -1$ **(f)** none of these.

(xi) ☐ If $f(x) = x^3 - 3x$ then the iterative formula for Newton's method becomes $x_{n+1} =$ **(a)** $x_n - x_n^3 - 3x_n$ **(b)** $x_n/(3x_n - 3)$ **(c)** $x_n - (x_n^3 - 3x_n)/(3x_n^2 - 3)$ **(d)** $x_n - (3x_n^2 - 3)/(x_n^3 - 3x_n)$ **(e)** $x_n - 3x_n^2 + 3$ **(f)** none of these.

(xii) ☐ Suppose $\int_1^2 f(t)dt = 1$. Then $\int_2^4 f(t/2)dt =$ **(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4 **(f)** none of these.

(xiii) ☐ The error formula for the left hand sum with n subintervals (for monotonic functions) is $E \le (b - a)|f(b) - f(a)|/n$. Using this formula, what is the minimum n needed to compute $\int_0^4 \sqrt{x^2 + 2x + 1}dx$ with error $\le .001$? **(a)** 2000 **(b)** 20,000 **(c)** 1,600 **(d)** 16,000 **(e)** 400 **(f)** 4,000

(xiv) ☐ Solve the intial value problem $y' = \sec^2(t)$ and $y(\frac{\pi}{4}) = 2$ **(a)** $y = \tan(t) - 1$ **(b)** $y = \cos^2(t) - 1/\sqrt{2}$ **(c)** $y = \tan(t) + 1$ **(d)** $y = \sin(1/t)$ **(e)** $y = \tan(t) + 1$ **(f)** none of these.

(xv) ☐ Simpson's approximation for $\int_0^\pi \sin(x)dx$ with $n = 6$ is (recall $y_k = f(x_k)$ where $x_k = a + k(b - a)/n$)
(a) $\frac{\pi}{6}[0 + 4\sin(\frac{\pi}{6}) + 2\sin(\frac{\pi}{3}) + 4\sin(\frac{\pi}{2}) + 2\sin(\frac{2\pi}{3}) + 4\sin(\frac{5\pi}{6}) + 0]$
(b) $\frac{\pi}{18}[0 + 4\sin(\frac{\pi}{6}) + 2\sin(\frac{\pi}{3}) + 4\sin(\frac{\pi}{2}) + 2\sin(\frac{2\pi}{3}) + 4\sin(\frac{5\pi}{6}) + 0]$
(c) $\frac{\pi}{18}[0 + 2\sin(\frac{\pi}{6}) + 4\sin(\frac{\pi}{3}) + 2\sin(\frac{\pi}{2}) + 4\sin(\frac{2\pi}{3}) + 2\sin(\frac{5\pi}{6}) + 0]$
(d) $\frac{\pi}{6}[0 + 2\sin(\frac{\pi}{6}) + 2\sin(\frac{\pi}{3}) + 2\sin(\frac{\pi}{2}) + 2\sin(\frac{2\pi}{3}) + 2\sin(\frac{5\pi}{6}) + 0]$
(e) $\frac{\pi}{6}[0 + \sin(\frac{\pi}{6}) + \sin(\frac{\pi}{3}) + \sin(\frac{\pi}{2}) + \sin(\frac{2\pi}{3}) + \sin(\frac{5\pi}{6}) + 0]$
(f) none of these.

(xvi) ☐ Let $F(t) = \int_0^t (1 + \sin^4(x))(1 - x)dx$ Then F takes its maximum value at **(a)** 1 **(b)** 0 **(c)** $\pi/2$ **(d)** π **(e)** 2π **(f)** none of these.

(xvii) ☐ Find the left hand sum approximation for $\int_{-1}^1 x^3$ with $n = 4$. **(a)** 1 **(b)** 1/2 **(c)** 0 **(d)** $-1/4$ **(e)** $-1/2$ **(f)** none of these.

(xviii) ☐ $E = \frac{1}{12}(b - a)^3 \frac{1}{n^2} M$ is the error term for **(a)** left hand sums **(b)** right hand sums **(c)** Simpson's rule **(d)** trapezoid rule **(e)** Calvaleri's rule **(f)** none of these.

(xix) ☐ The area of the region between the graphs of x^3 and x^4 with $0 \leq x \leq 1$ is **(a)** 1 **(b)** 1/2 **(c)** 1/4 **(d)** 1/20 **(e)** 1/24 **(f)** none of these.

(xx) ☐ What is the name of the following result: if f is continuous on an interval containing a and b and $f(a) < y < f(b)$ then there is a c with $a < c < b$ such that $f(c) = y$ **(a)** Rolle's theorem **(b)** mean value theorem **(c)** Riemann's theorem **(d)** min-max theorem **(e)** Simpson's rule **(f)** intermediate value theorem

2. **(1 pt each, 5 pts total)** Compute the derivative of the following functions.

(i) $f(x) = x^{1/2}+x^{-1/2}$,

$\frac{d}{dx}f(x) =$

(ii) $f(x) = (1+x^3)\sin(x)$,

$\frac{d}{dx}f(x) =$

(iii) $f(x) = \cos(x)/x$,

$\frac{d}{dx}f(x) =$

(iv) $f(x) = \tan(x^2)$,

$\frac{d}{dx}f(x) =$

(v) $f(x) = \sin^2(x)+\cos^2(x)$,

$\frac{d}{dx}f(x) =$

3. **(2 pts each, 10 pts total)** Find each of the following definite integrals (you need not simplify answers)

(i) $\int_{-1}^{1}(2x^4 - x^2 - 2)dx$,

(ii) $\int_0^{\pi/4}\sin(2x)dx$,

(iii) $\int_0^1(x^2+3)^5 x dx$,

(iv) $\int_0^\pi \sin^2(t)\cos(t)dt$,

(v) $\int_0^\pi \frac{\cos^2(x)-\sin^2(x)}{\cos(x)+\sin(x)}dx$,

4. **(3 pts each; 9 pts total)** In each of the following, give the answer as a definite integral. You SHOULD NOT evaluate the integral.

   (i) The region bounded by $y = x^3$, $y = 0$ and $x = 1$ is rotated around the x-axis to give a region in 3-space. Find the volume of the region.

   (ii) The region $0 \leq y \leq x^2 + \sin(x)$ with $1 \leq x \leq 2$ is rotated around the y-axis to give a region in 3-space. Find the volume of the region.

(iii) Find the arclength of the graph of $y = \sin(x)$ between $x = 0$ and $x = \pi$.

5. **(3 pts each, 6 pts total)** Evalute the following limits.

(i) $\lim_{a \to \infty} \int_1^a \frac{1}{x^3} dx$.

(ii) $\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} (\frac{k}{n})^2$ (Hint: use an integral)

6. **(10 pts)** A rocket in space starts at rest and is accelerated at 8 m/sec^2 for 100 seconds, coasts without acceleration for 50 seconds and then is accelerated at 16 m/sec^2 for 50 seconds after which it coasts without acceleration.

   Remember that acceleration is the derivative of velocity and that velocity is the derivative of position.

   (i) **(3 pts)** On the first graph, draw the rocket's velocity as a function of time as exactly as possible.

   (ii) **(3 pts)** On the second graph, sketch the distance traveled as a function of time. (Note, the vertical units are kilometers = 1000 meters).

   (iii) **(2 pts)** When is the speed of the rocket 1200 m/sec?

   (iv) **(2 pts)** When will the rocket be 500 kilometers from it starting point?