MIDTERM 2
MAT 142
11/11/05

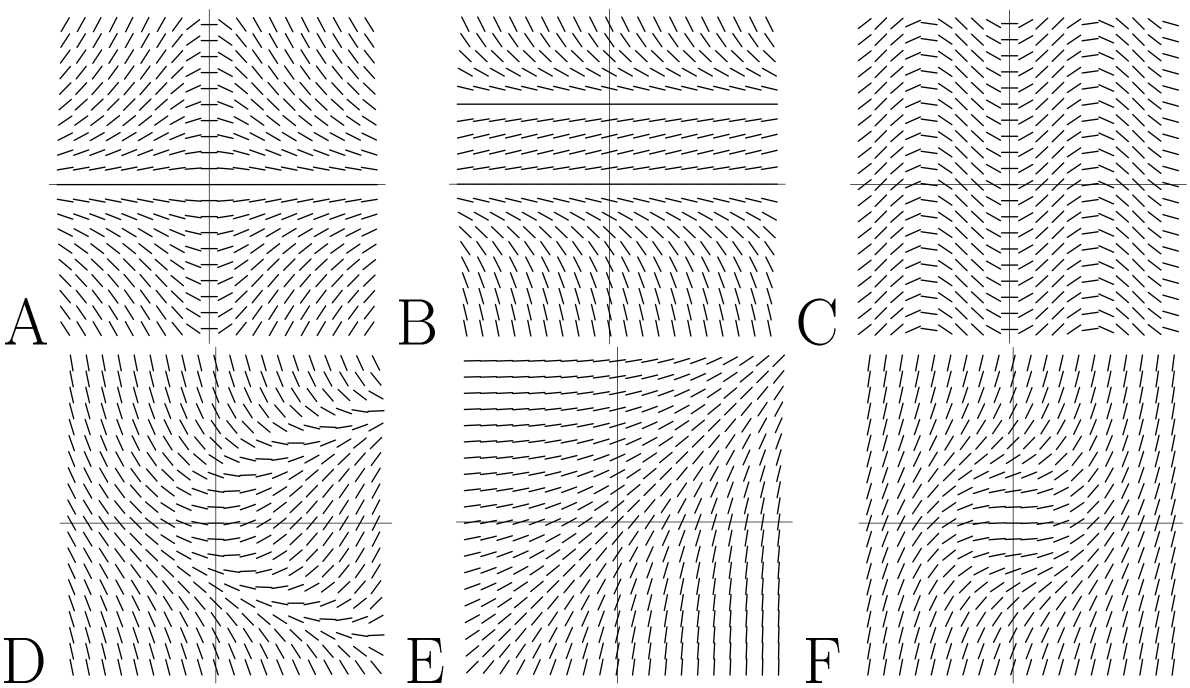
Name	Sec.

ID number	TA's name

1	2	3	4	5	total

THIS EXAM IS WORTH 50 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. NO NOTES OR CALCULATORS ALLOWED.

1. **(Part A, 6 pts)** Put the letter of the slope field in the box of the corresponding equation. All slope fields are graphed on $[-2,2]\times[-2,2]$.



i. ☐ $y' = x^2 + y^2$ **iii.** ☐ $y' = x - y^2$ **v.** ☐ $y' = (-2xy)/(1+x^2)$

ii. ☐ $y' = y(1-y)$ **iv.** ☐ $y' = e^{x-y}$ **vi.** ☐ $y' = \sin(3x)$

(Part B, 4 pts) Match each differential equation to the corresponding solution.

i. ☐ x^2 **iii.** ☐ $\sin(x)$

ii. ☐ e^{x^2} **iv.** ☐ $\cosh(x)$

A $y' + y = e^x$ **C** $xy' = 2y$

B $y'' = (2+4x^2)y$ **D** $y'' = -y$

2. **(2 pts each, 10 pts total)** Put a 'C' (for converges) or 'D' (for diverges) in the box next to each infinite series and explain why this is correct using tests from the textbook.

(a) ☐ $\sum_{n=1}^{\infty} \frac{1}{n+10}$

(b) ☐ $\sum_{n=1}^{\infty} \frac{n}{1+n^3}$

(c) ☐ $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$

(d) ☐ $\sum_{n=1}^{\infty} \frac{2^n}{n^n}$

(e) ☐ $\sum_{n=1}^{\infty} a_n$, where $a_1 = 1$ and $a_{n+1} = \frac{1}{2}(a_n + 1)$ for $n > 1$.

3. **(10 pts, 2 pts each)** Evaluate each of the following infinite series. Put the final answer in the box and show your work below.

(a) $\sum_{n=1}^{\infty} 3^n 4^{1-n} =$ $\boxed{}$

(b) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} =$ $\boxed{}$

(c) $1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots =$ $\boxed{}$

(d) $\sum_{n=0}^{\infty} \sin^n(x) =$ $\boxed{}$

(e) $\sum_{n=1}^{\infty}(1 + (-1)^n)2^{-n} =$ $\boxed{}$

4. **(10pts; 2pts each)** Solve each of the following differential equations.

(a) $\frac{dy}{dx} = e^{x-y}$

(b) $\frac{dy}{dx} = 2x\sqrt{1-y^2}, \qquad |y| < 1$

(c) $\frac{dy}{dx} = (1+y^2)e^x, \qquad y(0) = 0$

(d) $e^x \frac{dy}{dx} + 2e^x y = 1$

(e) $x\frac{dy}{dx} = x^2 + 3y$ with $y(1) = 1$, $x > 0$

5. **(5 pts each, 10 pts total)** Do TWO of the following problems (your choice). Put a mark in the box next to the two problems you want to be graded.

   (a) ☐ A tank contains 100 gallons of fresh water. A solution containing 1 lb/gal of soluble fertilizer runs into the tank at a rate of 1 gal/min and the well stirred mixture is drawn from the tank at a rate of 3 gal/min. Find the amount of fertilizer in the tank as a function of time.

   (b) ☐ Suppose Euler's method with $n = 100$ is applied to the differential equation $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 1$ to estimate $y(1)$. Is the resulting estimate y_{100} larger or smaller than the actual value $y(1)$ of the real solution? Explain why.

   (c) ☐ If $\{a_n\}$ is a positive sequence and if $\sum_{n=1}^{\infty} a_n$ converges then prove that $\sum_{n=1}^{\infty} (a_n)^2$ also converges.

   (d) ☐ Does the improper integral $\int_1^\infty \frac{|\sin x|}{x} dx$ diverge or converge? Justify your answer.