MIDTERM 1
MAT 141
10/13/00

Name	Sec.

ID number

1	2	3	4	total

THIS EXAM IS WORTH 40 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. YOU MAY USE THE BACKS OF PAGES FOR SCRATCH WORK.

1. **(1 pt each, 20 pts total)** Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

(i) ☐ Suppose $a < 0$ and $b > 0$. Then which of the following must be true?
(a) $ab > 0$ **(b)** $a - b > 0$ **(c)** $b - a > 0$ **(d)** $b^2 - a^2 > 0$ **(e)** $a^2 + b^2 < 0$ **(f)** none of these.

(ii) ☐ Suppose f is a linear function such that $f(1) = -1$ and $f(3) = 2$. Then $f(4) =$?
(a) 3 **(b)** $3\frac{1}{3}$ **(c)** $3\frac{2}{3}$ **(d)** $3\frac{1}{2}$ **(e)** $3\frac{3}{4}$ **(f)** none of these.

(iii) ☐ Which interval is the solution of $|x - 2| < 3$?
(a) $(0, 3)$ **(b)** $[-1, 3]$ **(c)** $(-2, 5)$ **(d)** $[0, 5]$ **(e)** $(-1, 5)$ **(f)** none of these.

(iv) ☐ Suppose f and g are given by the following tables. What is $f(g(2))$?

x	0	1	2	3	4
f(x)	0	4	3	2	1
g(x)	1	2	2	4	0

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4 **(f)** it is undefined.

(v) ☐ What is $\lim_{x\to 2} \frac{x^2-4}{x-2}$? **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** ∞ **(f)** none of these.

(vi) ☐ What is the natural domain of $\sqrt{x^2 - \frac{1}{x}}$? **(a)** $x > 0$ **(b)** $-1 < x < 1$ **(c)** $x > 1$ **(d)** $x \geq 1$ and $x < 0$ **(e)** $-1 < x < 0$ and $0 < x < 1$ **(f)** none of these.

(vii) ☐ Suppose that for all $C > 0$ there is a $\epsilon > 0$ so that $|x| < \epsilon$ implies $f(x) > C$. Then **(a)** $\lim_{x\to 0} f(x) = +\infty$ **(b)** $\lim_{x\to+\infty} f(x) = 0$ **(c)** $\lim_{x\to+\infty} f(x) = 1$ **(d)** $\lim_{x\to+\infty} f(x) = +\infty$ **(e)** $\lim_{x\to 0} f(x) = 0$. **(f)** none of these.

(viii) ☐ The derivative of g at x is defined to be
(a) $\lim_{h\to 0} \frac{g(x+h)-g(x)}{x}$ **(b)** $\lim_{h\to 0} \frac{g(h)-g(x)}{x+h}$ **(c)** $\lim_{h\to 0} \frac{g(x-h)-g(x)}{h}$ **(d)** $\lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$ **(e)** $\lim_{h\to 0} \frac{g(x+h)+g(x)}{h}$ **(f)** none of these.

(ix) Which of the following is true?
(a) If f has a limit at x_0 it is continuous at x_0. **(b)** If the left and right limits exist at x_0 then the limit exists at x_0. **(c)** If f is continuous at x_0 it has a limit at x_0. **(d)** If f is continuous at x_0 then it is differentiable at x_0. **(e)** If f is continuous at x_0 it is continuous on an interval around x_0. **(f)** none of these.

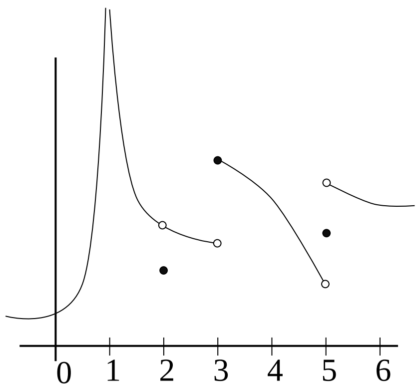
(x) Suppose f is continuous on the real line and $f(0) = 0$ and $f(10) = 2$. Then which of the following must be true?
(a) f attains a maximum which is > 2. **(b)** f takes the value 1 somewhere between 0 and 10. **(c)** f is increasing between 0 and 10. **(d)** f takes its maximum value between 0 and 10. **(e)** f is never negative. **(f)** none of these.

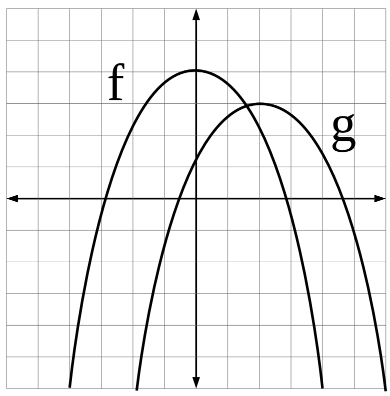
(xi) The derivative of $f(x) = x^2 + x^3$ at $x = 2$ is **(a)** 12 **(b)** 13 **(c)** 14 **(d)** 15 **(e)** 16 **(f)** none of these.

(xii) A car drives 30 miles at 60 mph and then another 50 miles at 50 mph. What is the average speed for the entire trip?
(a) 50 mph **(b)** $52\frac{1}{2}$ mph **(c)** $53\frac{1}{3}$ mph **(d)** 55 mph **(e)** 57 mph **(f)** none of these.

(xiii) List every point a between 0 and 6 in the graph on the left below where $\lim_{x \to a} f(x)$ does not exist (standard definition of finite limit)
(a) $1, 2, 3, 5$ **(b)** $1, 3, 5$ **(c)** $3, 5$ **(d)** $2, 5$ **(e)** $1, 5$ **(f)** none of these.

(xiv) Consider the graph on the left below. At what points does the function fail to be continuous? **(a)** $1, 2, 3, 5$ **(b)** $1, 3, 5$ **(c)** $3, 5$ **(d)** $2, 5$ **(e)** $1, 5$ **(f)** none of these.





(xv) What is the relationship between f and g in the graph on right above?
(a) $g(x) = f(x-1) + 2$ **(b)** $g(x) = f(x-2) + 1$ **(c)** $g(x) = f(x+2) - 1$ **(d)** $g(x) = f(x+1) - 2$ **(e)** $g(x) = f(x-2) - 1$ **(f)** none of these.

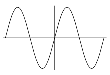
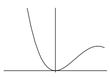
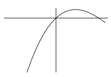
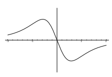
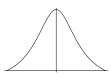
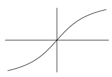
(xvi) Let $f(x) = |x^2 - 1|$. Then
(a) f is differentiable everywhere **(b)** f is not differentiable at $x = 0$ **(c)** f is not differentiable at $x = -1$ and $x = 1$ **(d)** f is not differentiable anywhere **(e)** f is not differentiable between -1 and 1 **(f)** none of these.

(xvii) The derivative of $(xh(x))^2$ is
(a) $1+2xh'(x)$ **(b)** $2xh'(x)(1+h'(x))$ **(c)** $2xh(x)(h(x)+xh'(x))$ **(d)** $h(x)+xh'(x)$ **(e)** $2xh(x)(1+xh'(x))$ **(f)** none of these.

(xviii) A dynamite blast blows a heavy rock straight up with an initial velocity of 160ft/sec. How high does the rock go (in feet)?
(a) 100 **(b)** 200 **(c)** 300 **(d)** 400 **(e)** 500 **(f)** none of these.

(xix) On planet X a ball dropped from rest falls 36 meters in 2 seconds. The acceleration due to gravity is
(a) 36m/sec^2 **(b)** 18m/sec^2 **(c)** 6m/sec^2 **(d)** $\sqrt{18}$m/sec^2 **(e)** 3m/sec^2 **(f)** none of these.

(xx) Which of the following satisfies $f'(0) < 0$?

(a)  **(b)**  **(c)**  **(d)**  **(e)**  **(f)** 

2. **(2 pts each, 10 pts total)** For each of the following functions, find the derivative function.

(i) $f(x) = x^6+x^{2/3}+x^{-2}$, $\frac{d}{dx}f(x) =$

(ii) $f(x) = x^3\cos(x)$, $\frac{d}{dx}f(x) =$

(iii) $f(x) = \sin(x^2+\tan(x))$,

$\frac{d}{dx}f(x) =$

(iv) $f(x) = (x^2-1)/(x-1)$,

$\frac{d}{dx}f(x) =$

(v) $f(x) = A\cos(Bx+C)$,

$\frac{d}{dx}f(x) =$

3. **(5 pts)** Compute the derivative of $f(x) = 1/x$ at $x = a \neq 0$ using only the limit definition of derivative.

4. **(5 pts)** Suppose that the functions f and g are defined on an open interval I containing the point x_0, that f is differentiable at x_0, that $f(x_0) = 0$ and that g is is continuous at x_0. Show the product fg is differentiable at x_0.