FINAL MAT 142 12/16/05

Name

Sec.

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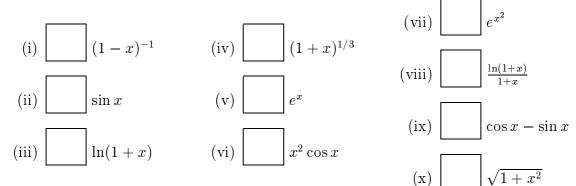
THIS EXAM IS WORTH 80 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. ADDITIONAL BLANK PAGES ARE PROVIDED FOR SCRATCH WORK. NO NOTES, BOOKS, CALCULATORS ARE ALLOWED.

- 1. (2 pts each, 40 pts total) Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.
 - (i) Simplify $\ln(x^2e^{2x})$ (a) $2\ln(x) + 2x$ (b) $4x\ln(x)$ (c) $2x\ln(x)$ (d) $2\ln(x) + x^2$ (e) $4x^2$ (f) none of these.
 - (ii) If $\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$ then $A = (a) \ 1 \ (b) \ 2 \ (c) \ 3 \ (d) \ 4 \ (e) \ 5 \ (f)$ none of these.
 - (iii) With the substitution $x = 3 \sin \theta$, the integral $\int \frac{x^2 dx}{\sqrt{9-x^2}}$ becomes (a) $9 \int \frac{d\theta}{\sin^3 \theta}$ (b) $9 \int \sin^2 \theta d\theta$ (c) $9 \int \sin^3 \theta d\theta$ (d) $27 \int (1-\sin \theta) d\theta$ (e) $27 \int \sin^3 \theta d\theta$ (f) none of these.
 - (iv) Which of the following improper integrals converges? (a) $\int_0^1 \frac{\ln x}{x} dx$ (b) $\int_0^1 x^{-3} dx$ (c) $\int_0^1 x^{-1/4} dx$ (d) $\int_1^\infty \frac{1}{\sqrt{x}} dx$ (e) $\int_1^\infty \frac{1}{x} dx$. (f) none of these.
 - (v) If the Taylor series $\sum_{n=1}^{\infty} c_n (x-2)^n$ converges at x=5, then it must also converge at (\mathbf{a}) -5 (b) -2 (c) -1 (d) 0 (e) 7 (f) none of these.
 - (vi) Evaluate $\int_0^2 \frac{2xdx}{x^2-5}$. (a) $\ln 2$ (b) $-\ln 2$ (c) 0 (d) $-\ln 5$ (e) $\ln 4$ (f) none of these.
 - (vii) What is the inverse of $y = e^{2x+1}$? (a) $y = \sqrt{\ln(x) 1}$ (b) $y = \frac{1}{2} \ln(x/e)$ (c) $y = \ln(\sqrt{x})$ (d) $y = \frac{1}{2}(\ln(x) + 1)$ (e) $y = \ln(2x + 1)$ (f) none of these.
 - (viii) The formula for Euler's method of solving y' = f(x, y) is (a) $y_{n+1} = y_n f(x_n, y_n) \Delta x$ (b) $y_{n+1} = y_n + f(x_n, y_n)$ (c) $y_{n+1} = y_n + f(x_n, y_n) \Delta x$ (d) $y_{n+1} = y_n f(x_n, y_n)$ (e) $y_{n+1} = y_n + f(x_n, y_n)(\Delta x)^2$ (f) none of these.
 - (ix) Find the solution of the differential equation $\frac{dy}{dx} = (1+y^2)e^x$. (a) $y = \tan x$ (b) $y = e^{\tan x}$ (c) $y = 1 + \tan^2 x$ (d) $y = e^x$ (e) $y = \tan(e^x)$ (f) none of these.

- (x) In an oil refinery, a tank contains 2000 gallons of gasoline that initially has 100 pounds of additive disolved in it. Gasoline containing 2 pounds of additive per gallon is pumped into the tank at 40 gal/min and the well mixed solution is pumped out at 45 gal/min. If y(t) is the amount of additive at time t then y satisfies which of the following differential equations: (a) $\frac{dy}{dt} = 40 \frac{45y}{2000-5t}$ (b) $\frac{dy}{dt} = 80 \frac{45y}{2000-5t}$ (c) $\frac{dy}{dt} = 40 \frac{45y}{2000-45t}$ (d) $\frac{dy}{dt} = 80 \frac{40y}{2000-45t}$ (e) $\frac{dy}{dt} = 40 \frac{40y}{2000-5t}$ (f) none of these.
- (xi) Which of the following is a true identity for hyperbolic trig functions?

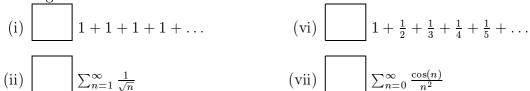
 (a) $\sinh 2x = 2 \sinh x \cosh x$ (b) $\sinh^2 x + \cosh^2 x = 1$ (c) $\cosh 2x = \cosh x + \sinh x$ (d) $\cosh^2 x = \sinh^2 x$ (e) $\sinh^2 x = \frac{1}{2} \cosh 2x$ (f) none of these.
- (xii) The improper integral $\int_1^\infty (1+x^s)^{-2} dx$ converges if and only if (a) s > 1 (b) s > 1/2 (c) s > 0 (d) s < -1 (e) s < 0 (f) none of these.
- (xiii) Which of the following is a geometric series? (a) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ (b) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ (c) $1 + \frac{1}{2} + \frac{1}{8} + \frac{6}{4} + \dots$ (d) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$ (e) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ (f) none of these.
- (xiv) The solution of the differential equation $y' = y^2 y 2$ with initial condition y(0) = 0 (a) is constant (b) decreases to -1 (c) decreases to $-\infty$ (d) increases to 2 (e) increases to $+\infty$ (f) none of these.
- (xv) The sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$ is (a) non-decreasing and convergent (b) non-increasing and convergent (c) non-decreasing and divergent (d) non-increasing and divergent (e) bounded and divergent (f) none of these.
- (xvi) Which of the following sequences is not bounded for n = 1, 2, 3, ...? (a) $\{n^2 2^{-n}\}$ (b) $\{\sqrt{n+1} \sqrt{n}\}$ (c) $\{n^2/(1+n^3)\}$ (d) $\{n/\ln n\}$ (e) $\{n^2/(1+n)^2\}$ (f) none of these.
- (xvii) Suppose $a_n = 2a_{n-1} + 1$. If $a_1 = 1$ then $a_5 =$ (a) 15 (b) 5 (c) 31 (d) 8 (e) 16 (f) none of these.
- (xviii) The series $\sum_{n=1}^{\infty} (2x+1)^n$ converges exactly for (a) all x (b) -1 < x < 0 (c) -1 < x < 1 (d) 0 < x < 1 (e) 0 < x < 1/2 (f) none of these.
 - (xix) The root test says that if $a_n \ge 0$ then $\sum_{n=0}^{\infty} a_n$ converges if (a) $\lim_n a_{n+1}/a_n < 1$ (b) $\lim_n a_n < 1$ (c) $\lim_n (a_n)^{1/n} < 1$ (d) $\lim_n a_n/a_{n+1} < 1$ (e) $\lim_n na_n < 1$ (f) none of these.
 - (xx) Evalute the improper integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. (a) π (b) $\pi/2$ (c) 1 (d) 2π (e) 2 (f) none of these.

2. (1 pt each, 10 pts total) Match each function with its Taylor series expan	sion
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A $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \dots$	$\mathbf{N} \ 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \dots$
B $2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \dots$	$\mathbf{O} \ x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + \dots$
$\mathbf{C} \ x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$	$\mathbf{P} \ x^2 - \frac{1}{2}x^4 + \frac{6}{24}x^6 - \dots$
$\mathbf{D} \ 1 + x^2 + x^4 + x^6 + \dots$	$\mathbf{Q} \ 1 + x^{2} + \frac{1}{2}x^{4} + \frac{1}{6}x^{6} + \dots$
E $1 + \frac{1}{3}x - \frac{2}{9}x^2 + \frac{10}{27}x^3 - \frac{80}{81}x^4 + \dots$	$\mathbf{R} \ 1 + \frac{1}{3}x + \frac{1}{9}x^2 + \frac{1}{27}x^3 + \dots$
F $1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 - \dots$	$\mathbf{S} \ 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \dots$
G $1 + \frac{1}{2}x^2 - \frac{1}{4}x^4 + \frac{3}{8}x^6 - \frac{15}{16}x^8 + \dots$	$T 1 - x + x^2 - x^3 + x^4 - \dots$
$\mathbf{H} \ x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \dots$	$\mathbf{U} \ x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$
I $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$	$\mathbf{V} \ 1 - \frac{1}{3}x^3 + \frac{1}{4}x^4 - \frac{1}{5}x^5 + \dots$
$\mathbf{J} \ 1 + x + x^2 + x^3 + x^4 + \dots$	$\mathbf{W} \ x + x^3 + x^5 + x^7 + \dots$
K $1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{60}x^5 + \dots$	$\mathbf{X} \ 1 + \frac{1}{4}x^2 + \frac{1}{27}x^3 + \frac{1}{256}x^4 + \dots$
$\mathbf{L} \ x^2 - \frac{1}{3}x^6 + \frac{1}{120}x^{10} - \dots$	$\mathbf{Y} \ 1 + \frac{4}{2}x + \frac{1}{4}x^2 + \frac{3}{8}x^3 + \dots$
$\mathbf{M} \ x - \frac{9}{6}x^3 + \frac{17}{120}x^5 + \dots$	Z none of these

3. (1 pt each, 10 pts total) Label each series as either A (Absolutely convergent), or C (Conditionally convergent or D (Divergent). each of the following infinite series converges or diverges.

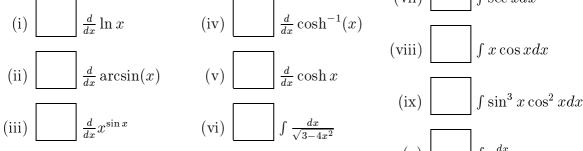


(iii)
$$1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \dots$$
 (viii) $\sum_{n=0}^{\infty} \frac{\cos(\pi n)}{n}$

(iv)
$$\sum_{n=1}^{\infty} n^2 2^{-n}$$
 (ix)
$$\sum_{n=1}^{\infty} (\ln n)^{-2}$$

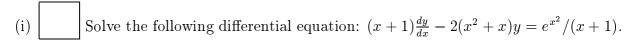
(v)
$$\sum_{n=1}^{\infty} \frac{n}{n^n}$$
 (x)
$$\sum_{n=0}^{\infty} \sin(n)$$

4. (1 pt each, 10 pts total) Eval of the correct answer in the box	or find some inte	egral and put the letter
	(vii)	$\int \sec x dx$



$\mathbf{A} \ x^{\sin x} (\frac{1}{x} \sin x + \ln x \cos x)$	$N 1/(x \sqrt{x^2-1})$
$\mathbf{B} \frac{1}{1-x^2}$	$O_{\frac{1}{5}}\cos^5 x - \frac{1}{3}\cos^3 x$
$\mathbf{C} \stackrel{1}{\stackrel{1}{x}}$	$\mathbf{P} \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x$
$\mathbf{D} - \sinh x$	$\mathbf{Q} \cos^5 x - \sin^3 x$
$\mathbf{E} \tan(x)$	$\mathbf{R} \cosh x$
$\mathbf{F} x^{\sin x} \ln x \cos x$	$\mathbf{S} \cot x$
$\mathbf{G} e^x$	${f T} anh x$
$\mathbf{H} \ln \sec x + \tan x $	$\mathbf{U} \frac{1}{\sqrt{r^2-1}}$
$\mathbf{I} x \cos x + \sin x$	$\mathbf{V} \arcsin(x)$
$\mathbf{J} \frac{1}{2} \arcsin(\frac{2x}{\sqrt{3}})$	$\mathbf{W} x^{\cos x}$
$\mathbf{K} x \ln x$	$\mathbf{X} x \sin x + \cos x$
$\mathbf{L} \ln \cos x + \tan x $	$\mathbf{Y} \sinh x$
$M 1/\sqrt{1-x^2}$	Z none of these

5. (5 pts each, 10 pts total) Do TWO of the following problems (your choice). Put a mark in the box next to the two problems you want to be graded. Put your work on the following pages and clearly mark which problem you are doing.



- (ii) State Taylor's theorem (or the remainder estimation theorem) and use it to prove that the Maclaurin series for $\sinh x$ converges to $\sinh x$ for all real numbers.
- (iii) Find a power series solution (up to and including the x^4 term) to the differential equation y'' y' y = 0 with y'(0) = 1 and y(0) = 1
- (iv) Give an example of a series so that $\sum_{n=1}^{\infty} a_n$ converges but such that $\sum_{n=1}^{\infty} (a_n)^2$ diverges.
- (v) Prove $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ is irrational.