MAT 126 Fall 2020, Trigonometric integration strategies:

To integrate $\int \cos^j x \sin^k x dx$**:**

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

To integrate products of $\sin(ax), \sin(bx), \cos(ax), \cos(bx)$ **use:**

(d) $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

To integrate $\int \tan^k x \sec^j x dx$**:**

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$. Then use $u = \sec x$

(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.

(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Additional formulas:

$\int \sec^2 x dx = \tan x + C$

$\int \sec x \tan x dx = \sec x + C$

$\int \tan x dx = \ln|\sec x| + C$

$\int \sec x dx = \ln|\sec x + \tan x| + C$

$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx$

$\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx$

$\int \sec^3 dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C.$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = 2\tan x/(1 - \tan^2 x)$

$\sin(x+y) = \sin x \cos y + \cos x \sin y.$

$\cos(x+y) = \cos x \cos y - \sin x \sin y.$

$\tan(x+y) = (\tan x + \tan y)/(1 - \tan x \tan y).$

$\sin^2(x/2) = (1 - \cos x)/2$

$\cos^2(x/2) = (1 + \cos x)/2$