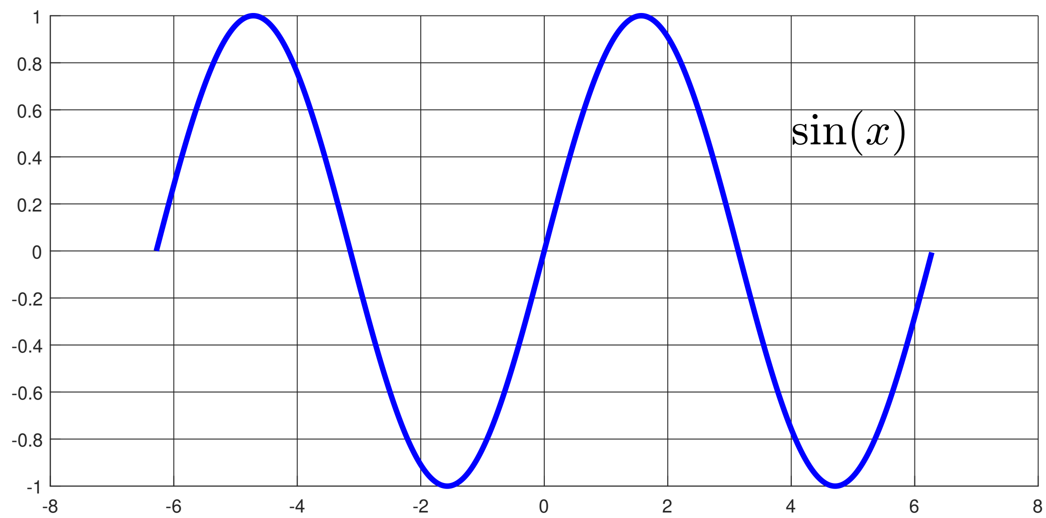
MAT 126.01, Prof. Bishop, Thursday, Sept. 17, 2020

Thursday, September 17, 2020
Section 1.7, Integrals resulting in Inverse Trig Functions.

- Derivatives of inverse functions
- Sin and its inverse
- Tan and its inverse
- Sec and its inverse
- Examples



1
0.8
0.6
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-0.2
-0.4
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-0.8
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-8
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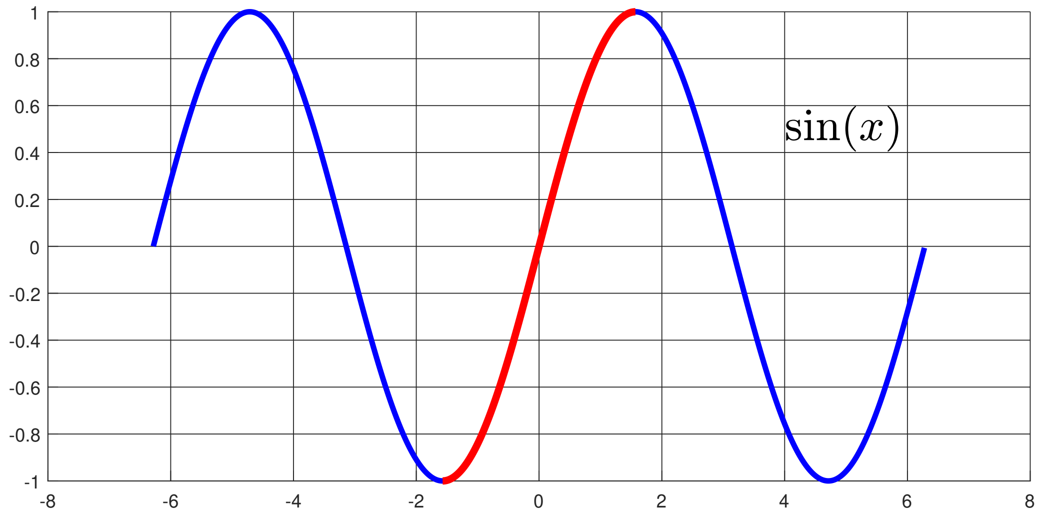
0
2
4
6
8
$\sin(x)$



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-8
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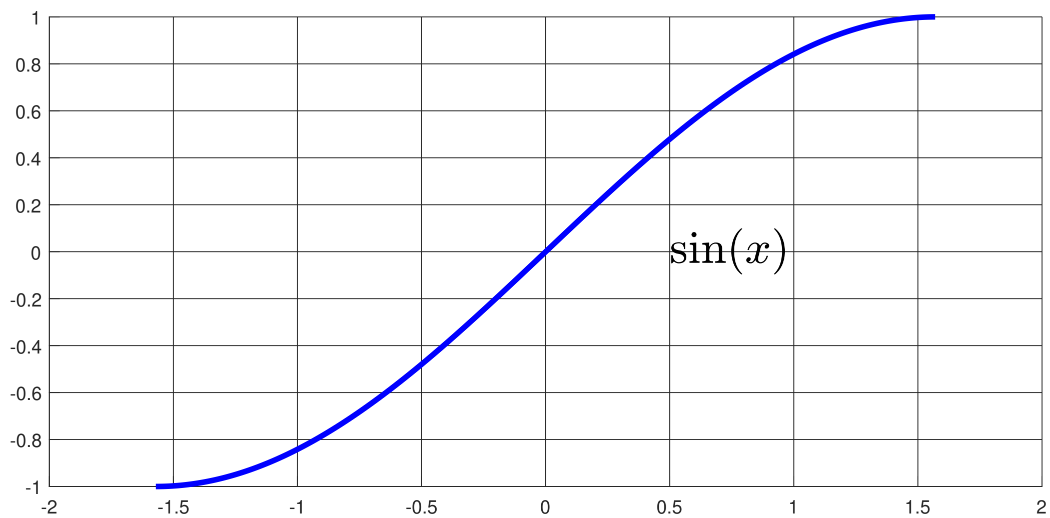
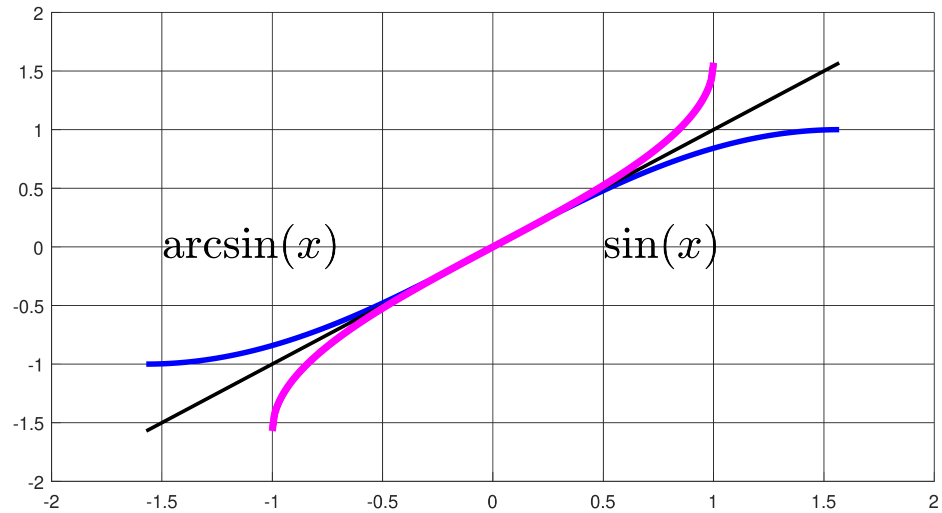
sin(x)



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
sin(x)



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
arcsin(x)
sin(x)

Functions f and g are inverses if $f(g(x)) = x$.

Examples e^x and $\ln x$

Examples x^2 (for $x > 0$) and $\sqrt{x}$

A function must be 1-to-1 to have an inverse

Many common functions have to restricted to have an inverse. Like x^2.

Graph of inverse is reflection of graph of f over diagonal $y = x$

If f and g are inverse functions then

$$g'(x) = \frac{1}{f'(g(x))}$$

Derive using chain rule.

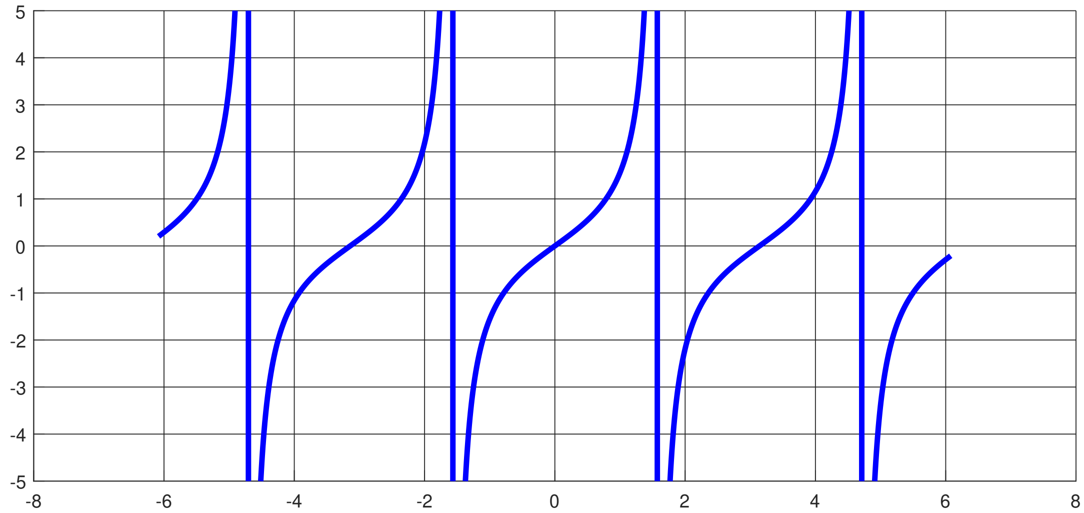
Apply to $\sin(x/a)$ and $a\sin^{-1}(x)$.

Derive $\frac{d}{dx}\sin^{-1}(x/a) = \frac{1}{\sqrt{a^2-x^2}}$

Thus

$$\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1}(x/a) + C$$

Find $\int_0^3 \frac{1}{\sqrt{9-x^2}} dx$


5
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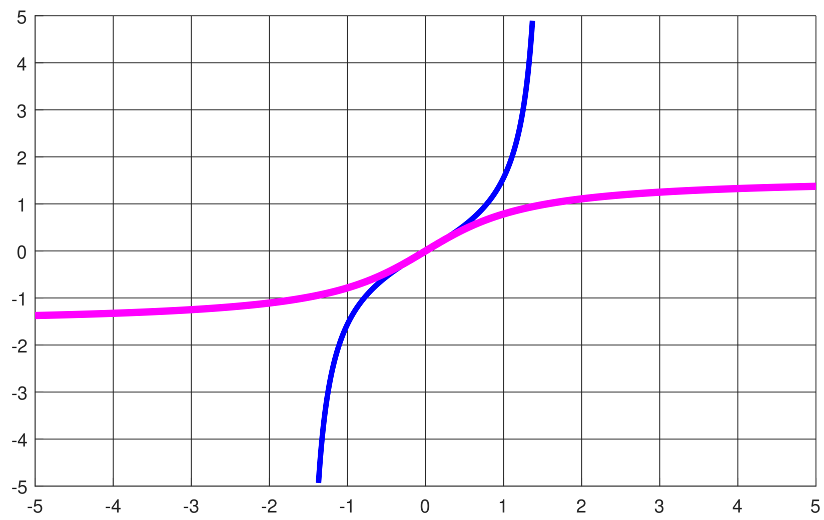
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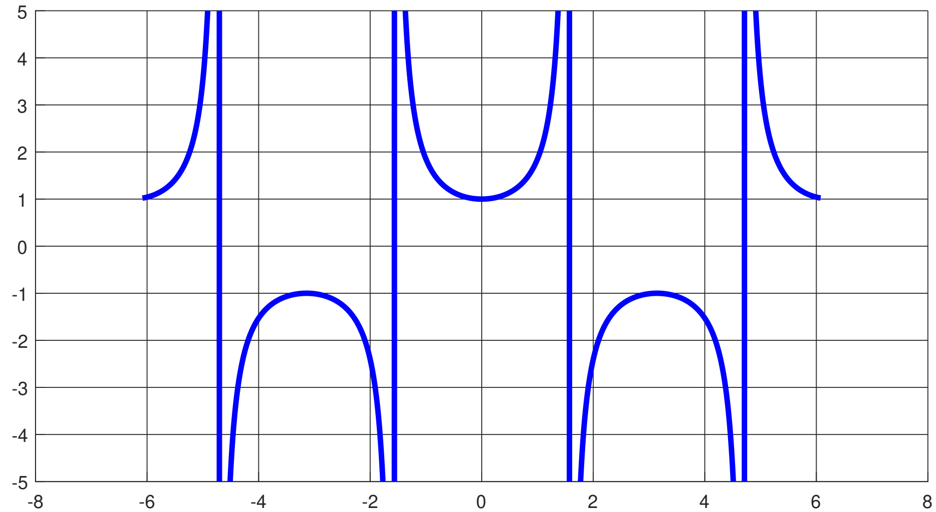
-4
-3
-2
-1
0
1
2
3
4
5

Derive

$$\frac{d}{dx}\tan^{-1}(x/a) = \frac{a}{a^2+x^2}$$

$$\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}(x/a) + C$$



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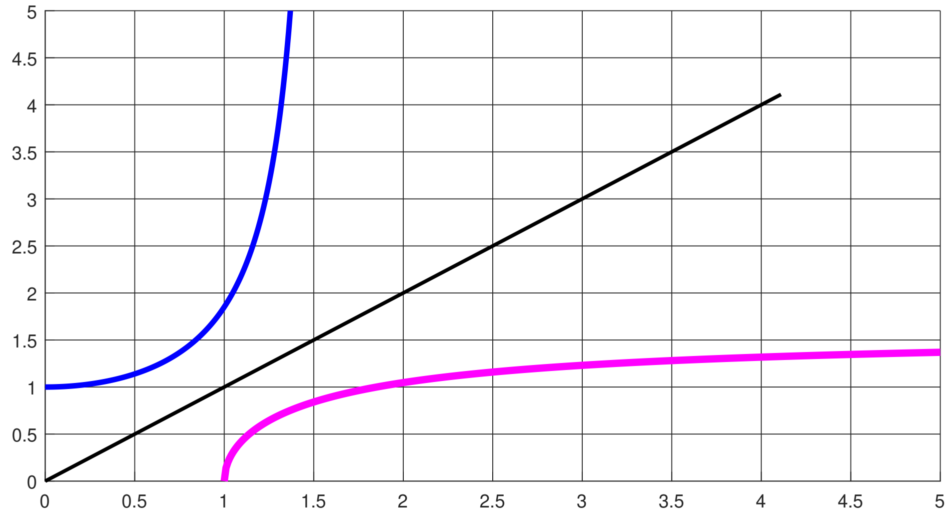

0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Derive

$$\frac{d}{dx}\sec^{-1}(x/a) = \frac{a}{x\sqrt{x^2 - a^2}}$$

$$\int \frac{a}{x\sqrt{a^2 - x^2}} = \frac{1}{a}\sec^{-1}(x/a) + C$$

Find $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

Find $\int \frac{dx}{\sqrt{4-9x^2}}$.

Find $\int \frac{dx}{\sqrt{x^2+9}}$.

Find $\int_0^2 \frac{dx}{\sqrt{x^2+4}}$.