MAT 126.01, Prof. Bishop, Tuesday, Sept. 15, 2020

Tuesday, September 15, 2020
Finish Section 1.5, Substitution
Section 1.6, Substitution

- Using trig substitutions with substitution
- Definition of natural logarithm
- Other logarithms
- Differentiation of logartiths
- Definition of natural exponents
- Other bases
- Differentiation of exponentials
- Examples

next midterm

Sometimes some algebra or trig identites are helpful:

Find $\int \cos^3(x)dx$.

TRICK

$\int \sin^3$

$$\boxed{\cos^2 + \sin^2 = 1}$$

$$\cos^2 = 1 - \sin^2$$

$$\int \cos^3 = \int \cos \cdot \cos^2 = \int \cos(1-\sin^2)$$

$$= \int \cos - \cos \sin^2$$

$$\int \cos = \sin$$

$u = \sin x \quad du = \cos x\, dx$

$$\int \cos x \sin^2 x\, dx = \int u^2 du$$

$$= \frac{1}{3}u^3$$

$$\boxed{\sin x - \frac{1}{3}\sin^3 x + C}$$

Sometimes some algebra or trig identites are helpful:

Find $\int_0^{\pi} \sin^2(x)dx$. $\stackrel{?}{=}$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$= \int \frac{1}{2}(1 - \cos 2x)\,dx$$

$u = 2x$

$du = 2\,dx$

$$= \int \frac{1}{2} - \int \frac{1}{2}\cos 2x\,dx -$$

$$= \frac{1}{2}x - \frac{1}{4}\int \cancel{\cos 2x}\ \underbrace{2\,dx}_{du}$$

$$\int \cos u\,du$$

$$-\frac{1}{4}\sin u$$

$$= \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

We define

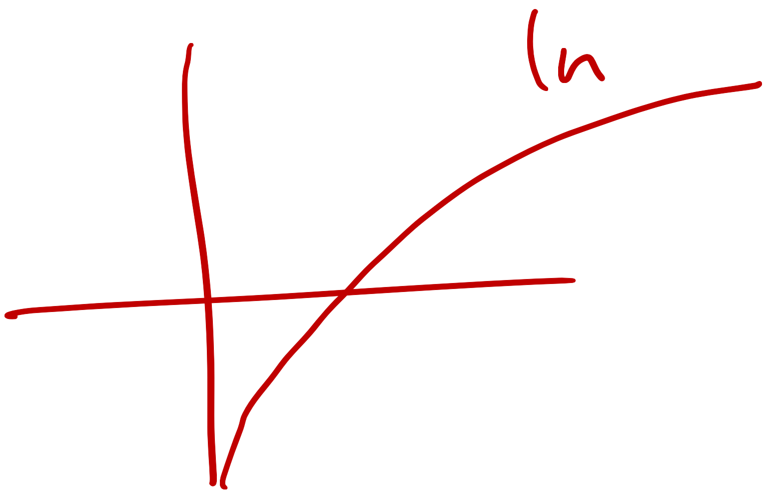
$$\boxed{\ln(x) = \int_1^x \frac{1}{t}\,dt.} = \int_1^x t^{-1}\,dt$$

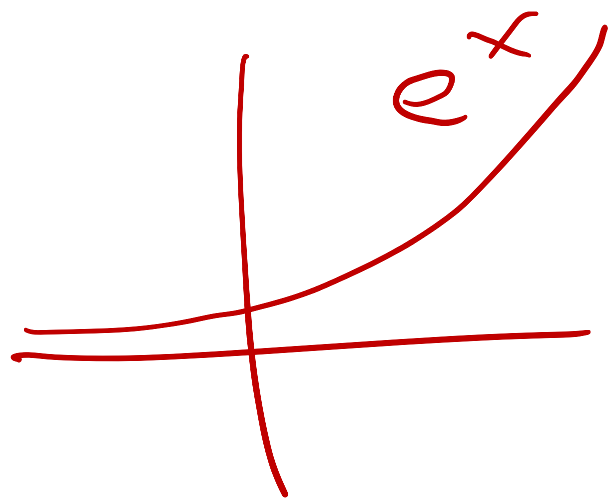
$\ln x$, e^x

$\int x^n = \frac{1}{n+1} x^{n+1}$

$n=-1$

$\frac{1}{0} x^0$?





Then $\frac{d}{dx}\ln x = \frac{1}{x}$ is obvious.

$$\ln x = \int_1^x \frac{1}{t}\,dt$$

$$(\ln x)' = \frac{1}{x} \quad \checkmark$$

Derive $\ln(1) = 0$

$$\ln(1) = \int_1^1 \frac{1}{t}\, dt$$

$$= 0$$

Derive $\ln(\frac{1}{x}) = -\ln(x)$

$$\ln\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{1}{xt}\, x\,dt$$

$u = xt \qquad du = x\,dt$

$t = 1 \quad u = x$

$t = 1/x \quad \to \quad u = 1$

$$= \int_x^1 \frac{1}{u}\,du$$

$$= -\int_1^x \frac{1}{u}\,du = -\ln(x)$$

Derive $\underline{\ln(xy) = \ln(x) + \ln(y)}$

Derive $\ln(x^p) = p\ln(x)$

Define $\log_b x = \ln(x)/\ln(b)$.

$$\log_{10} x = \frac{\ln x}{\ln 10}$$

$$\frac{d}{dx}\log_b x = \frac{1}{x \ln b}\,.$$

$$(\log_b x)' = \left(\frac{\ln x}{\ln b}\right)' = \frac{1/x}{\ln b}$$

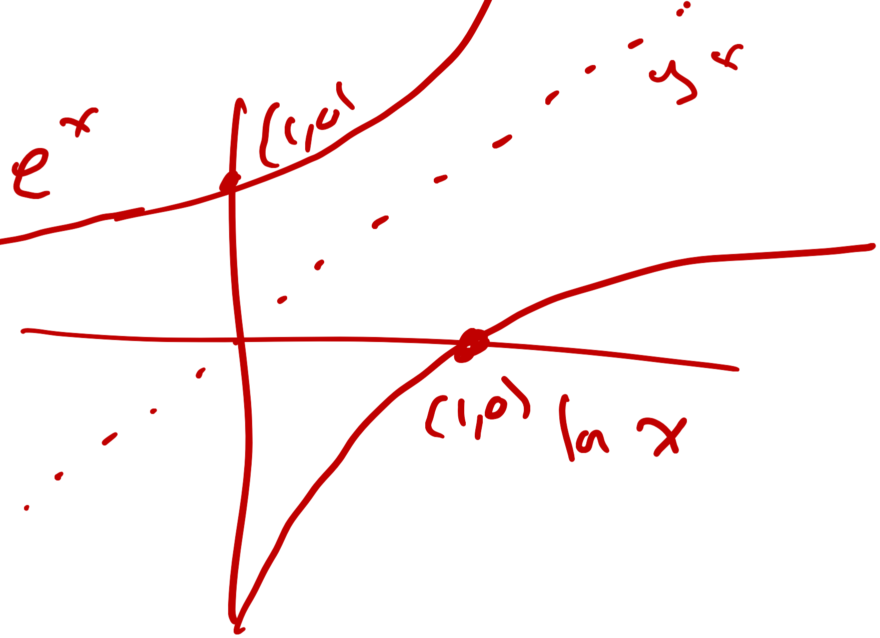
Derive $\underline{\int \ln x dx} = \underline{x \ln x - x} + C = x(\ln x - 1) = C$

$$(x \ln x - x)' = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1$$

$$= \ln x \quad \cancel{+1} \quad \cancel{-1}$$

Derive $\int \log_a x dx = \frac{x}{\log a} x(\ln x - 1) = C$

Define e^x as the inverse of $\ln x$.



$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

f, g inverses $\quad f(g(x)) = x$

$\sqrt{x}, \quad x^2$

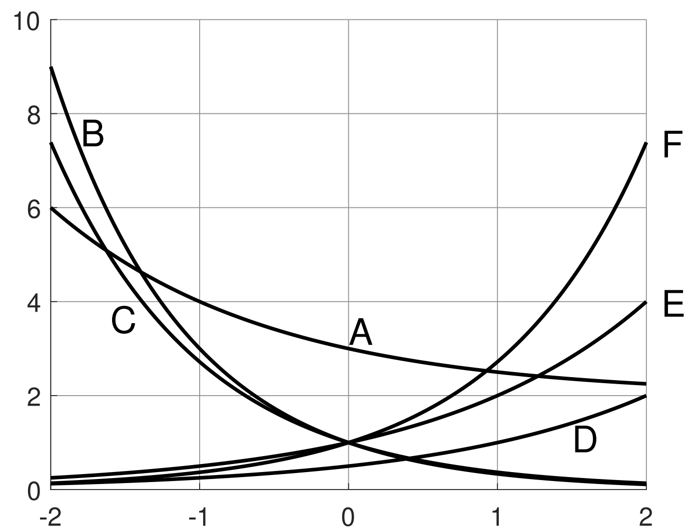
Derive $\frac{d}{dx}e^x = e^x$.

Define $a^x = e^{x \ln a)}$

Derive $\frac{d}{dx}a^x = a^x \ln x$.

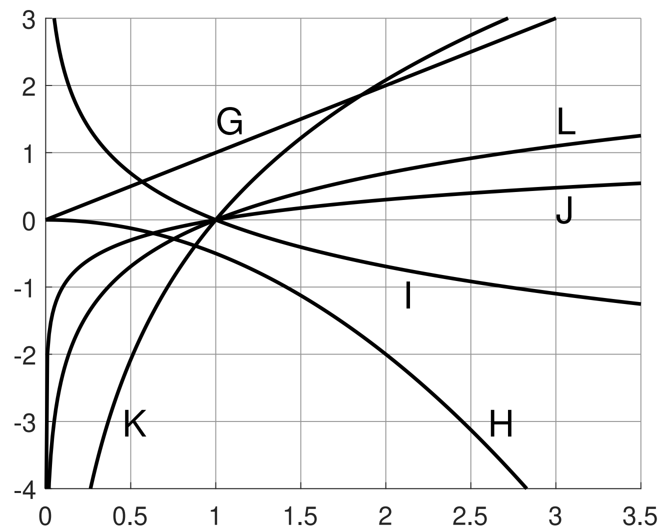
Find the graph of e^x.

Find the graph of 3^{-x}.



Find the graph of $\ln x$.

Find the graph of $\ln \frac{1}{x}$.



Find $\int \frac{2x+3}{x^2+3x+4}dx$

Find $\int_0^{\pi/2} \frac{\sin x}{\cos x+1} dx$

Find $\int e^x \sqrt{1+e^x} dx$

Find $\int \frac{1}{x \ln x} dx$

Quiz 3

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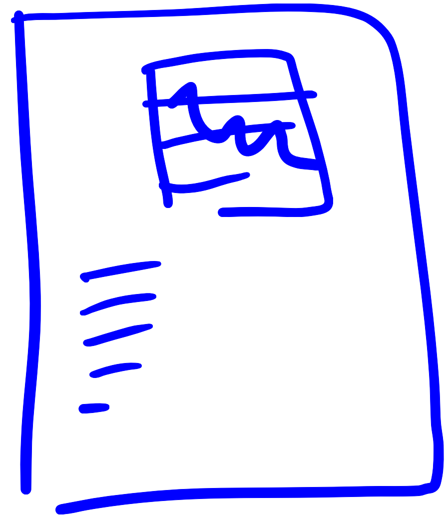
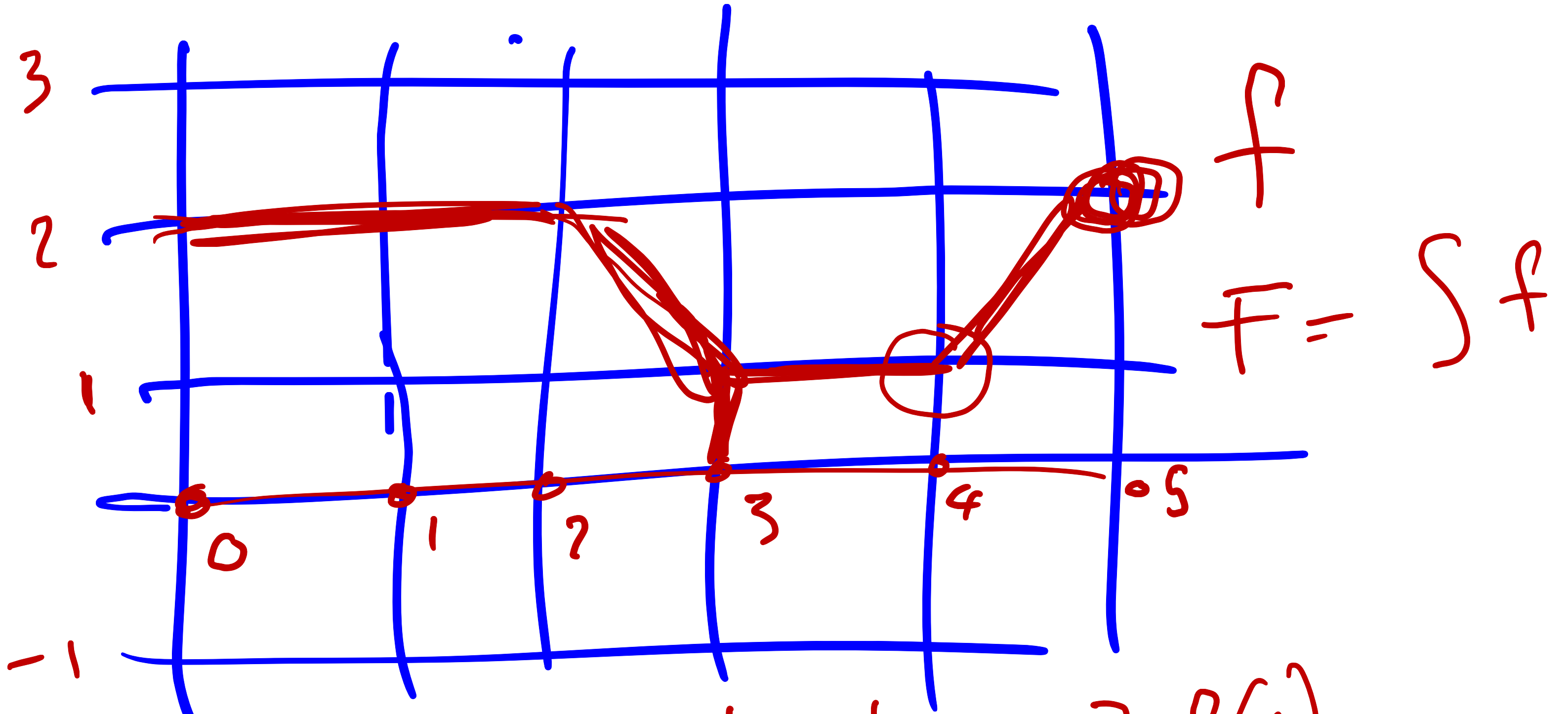




$F(3) = ?$ $F'(4) = ?$ $f(4)$

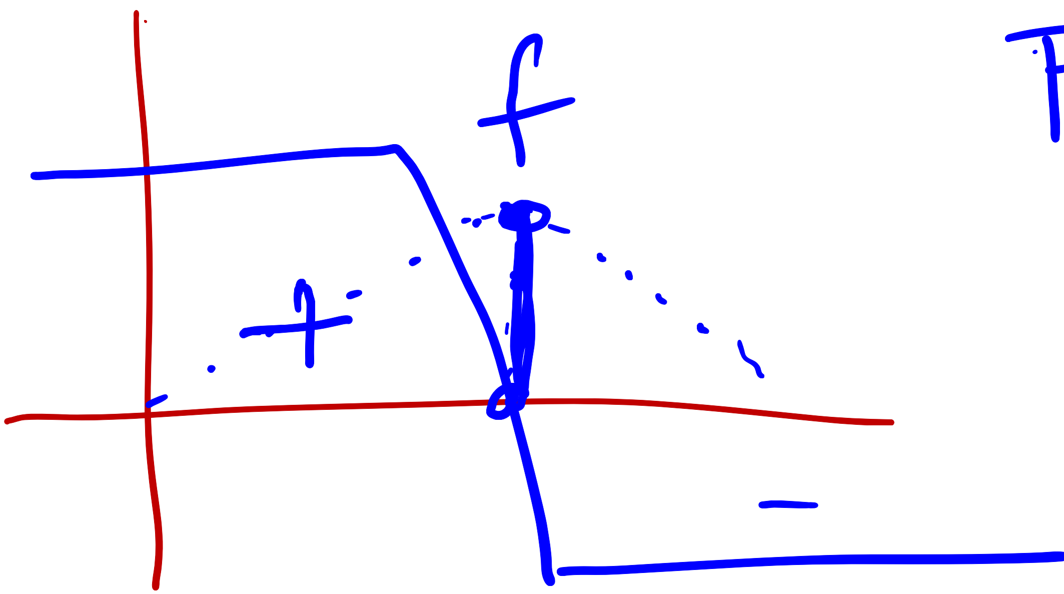
where is F concave? $F' > 0$ $x = 5$

Where does F take its max?

(what x value). $x=5$

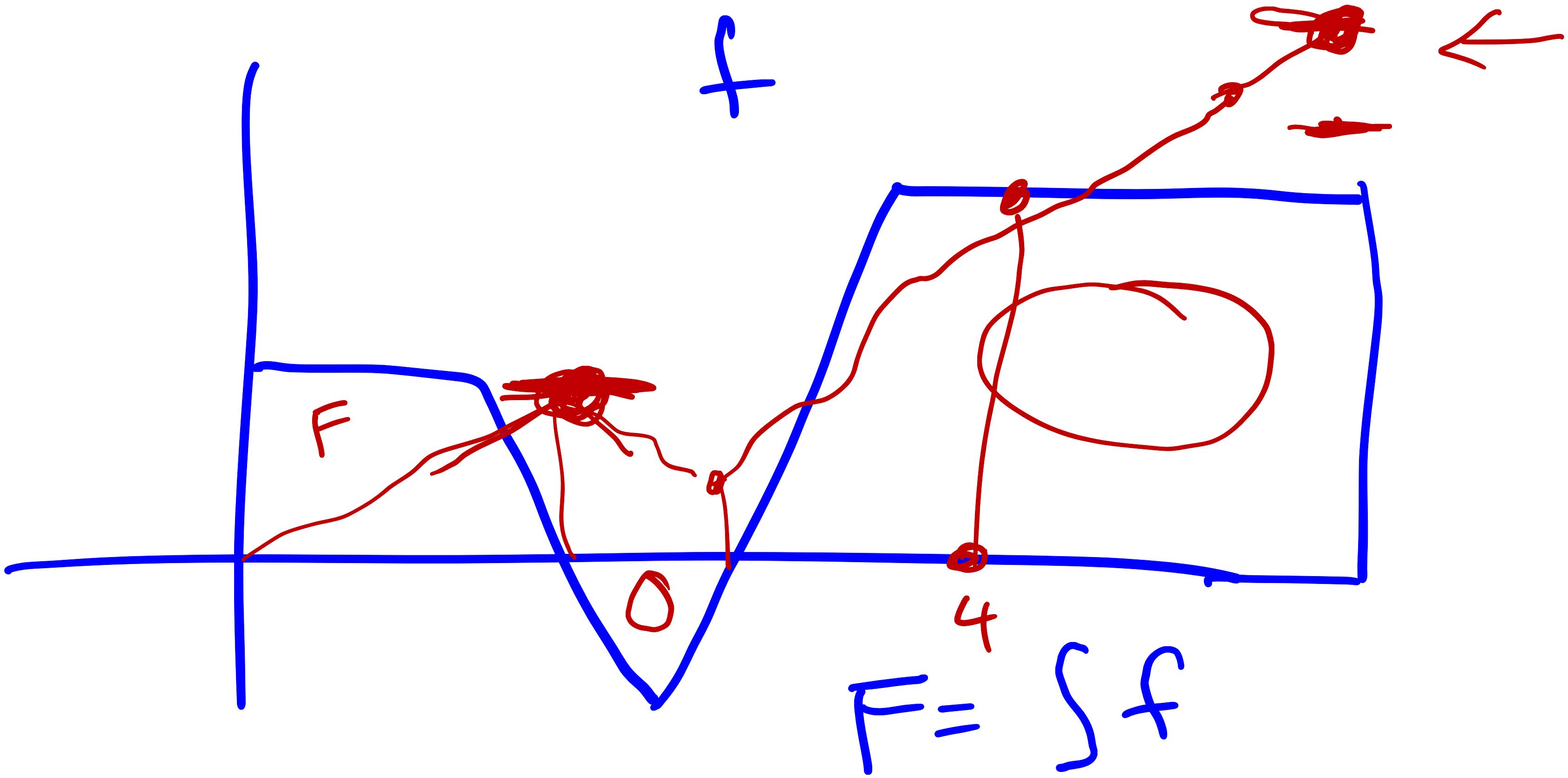
What is max value? $F(5) = \int_0^5 f$

$= 8$



$F = \int f$

$F' = f$



$$a(x) = \int_0^{x^2} f(t)\,dt$$

$$a(x) = F(x^2)$$

$$a'(x) = F'(x^2)\cdot 2x$$

$$a'(2) = F'(4)\cdot 4 = f(4)\cdot 4$$

$$P(t) = \int_0^t v(s)\,ds$$

$$= \int_0^t 30 - 20s\,ds$$

$\int_a^b f$

$= F(b) - F(a)$

$$= 30s - 10s^2 \Big|_0^t$$

$$= \left(30t - 10t^2\right) - (0$$

$v = P'$ $\quad a' = v' = P''$

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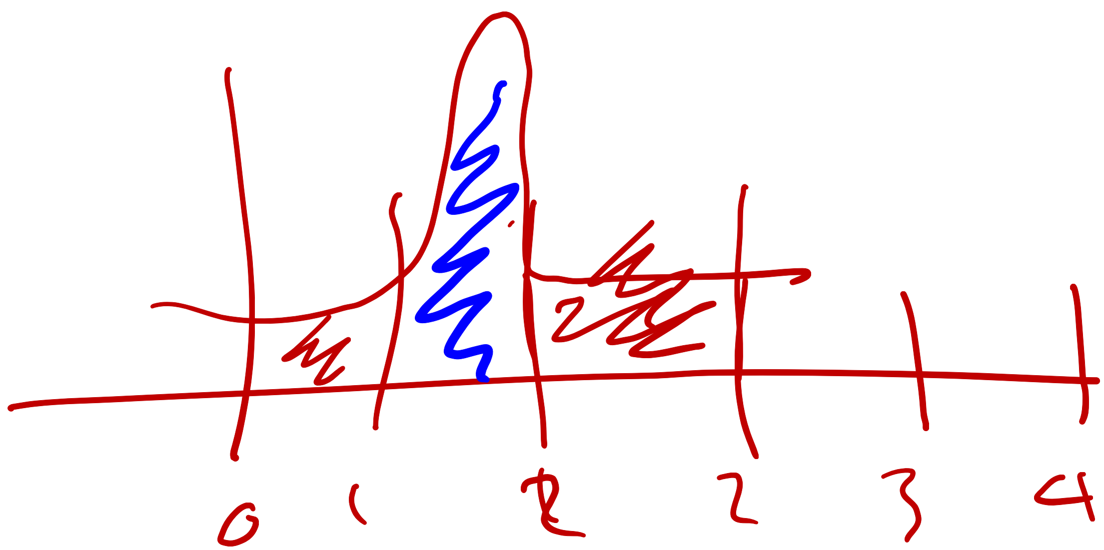
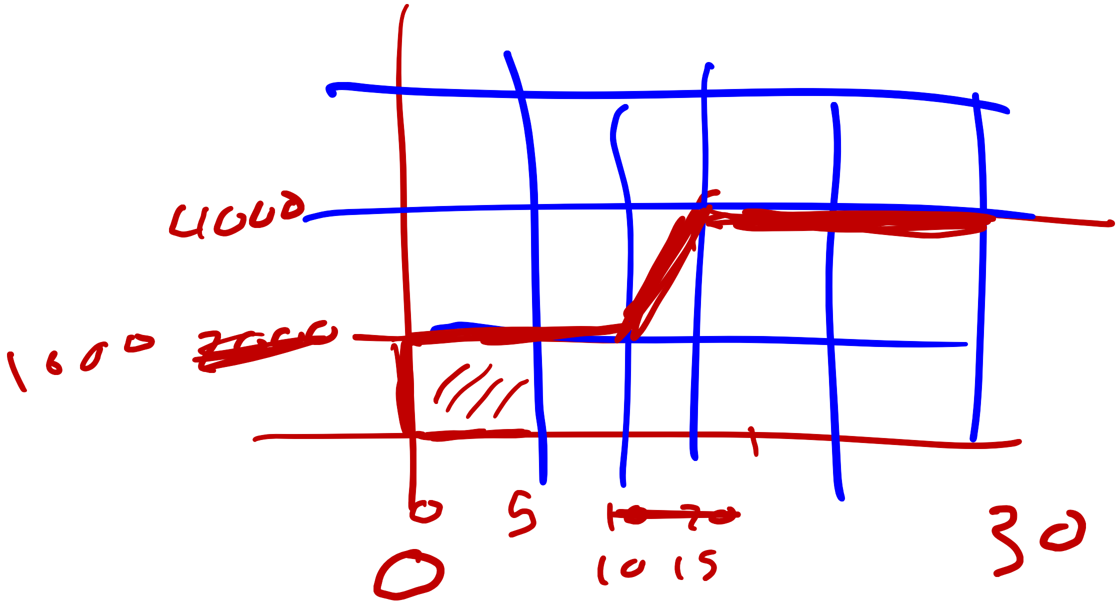


$\int_0^1 f$

$\int_1^2 f$

$\int_2^2$

8



$2

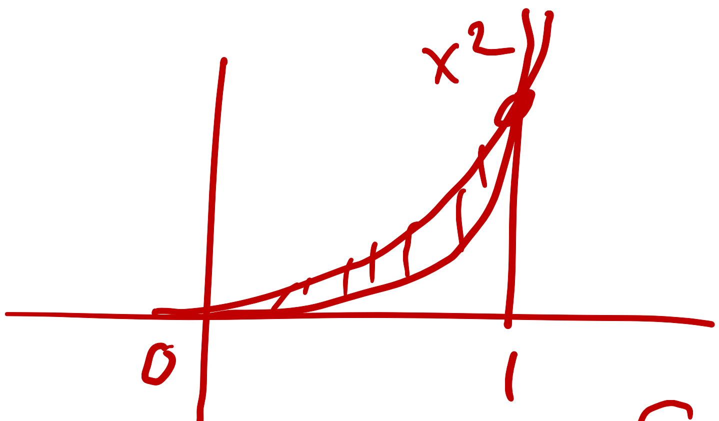
4

$3

5,000 × area

× $2

9



x^2, x^3

f, g

area between

$$\int_0^1 x^2 - x^3\,dx.$$

f − g

(10) $u = x^3 + 1$ lets you evaluate:

$\int \sin(\cancel{x^3+1})\, dx$?

$du = 3x^2$

$\frac{1}{3}\int \cos(\underbrace{x^3+1})\, \underbrace{3x^2\, dx}$ ✓

$\int e^{\cancel{x^3+1}}\, ?\, dx$

office hours start

~ 11:30 today