

MAT 126.01, Prof. Bishop, Thursday, Sept. 10, 2020

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Section 1.5, Substitution

- ▶ Recall the chain rule for derivatives.
- ▶ Substitution rule for integrals (indefinite integrals).
- ▶ Substitution rule for integrals (definite integrals).
- ▶ Polynomial examples
- ▶ Trigonometric examples
 - ▷ Review common trig formulas
 - ▷ Using trig identities to simplify integrals.

Substitution Rule:

Suppose f, g, g' are continuous. The

$$\int f(g(x))g'(x)dx = \int (f(g(x)))'dx = f(g(x)) + C.$$

$$\int \sin^2(x) \cos(x)dx = \int [\sin(x)]^2 \cos(x)dx$$

Example: Let $u = \sin(x)$, so $\frac{du}{dx} = \cos(x)$ or $du = \cos(x)dx$.

$$\int \sin^2(x) \cos(x)dx = \int [u]^2 du = \frac{1}{3}u^3 + C$$

Sometimes we need to multiply and divide by a factor to get du correct.

Find $\int \sqrt{3x + 2} dx$

Find $\int x \sin(x^2) dx$

Find $\int e^{4x} dx$

Find $\int xe^{x^2} dx$

Find $\int (x + 1) \cos(x^2 + 2x + 1) dx$

Find $\int \sin^{10}(x) \cos(x) dx$

Using substitution with definite integrals is a little trickier. You you also have to change the limits of integration:

Find $\int_0^1 x \sin(\pi x^2) dx$.

Find $\int_1^2 \frac{2 \ln(x^2+1)}{x^2+1} dx$.

Find $\int_2^4 \frac{x}{\sqrt{x-1}} dx$.

Find $\int_1^2 \frac{2 \ln(x^2+1)}{x^2+1} dx$.

Sometimes some algebra or trig identities are helpful:

Find $\int \cos^3(x) dx$.

Sometimes some algebra or trig identities are helpful:

Find $\int_0^\pi \sin^2(x) dx$.

