MAT 126.01, Prof. Bishop, Tuesday, Oct 6, 2020
Some HW 6 problems
Section 2.4, Arc length and surface area

$a = 2 \quad b = 7$

HW 6, Prob 6, Part A:

Compute the area between the parabola $y = ax - bx^2$ and the x-axis.

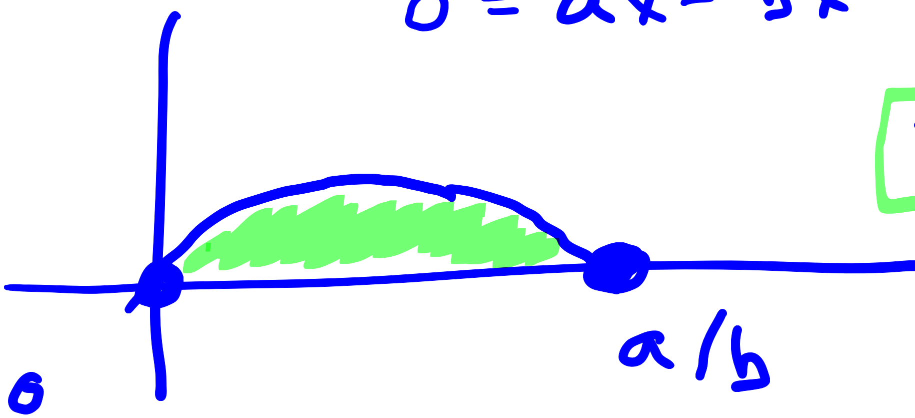
$$0 = ax - bx^2 = x(a - bx)$$

$\boxed{x = 0}$

$a - bx = 0$

$a = bx$

$\boxed{\frac{a}{b} = x}$



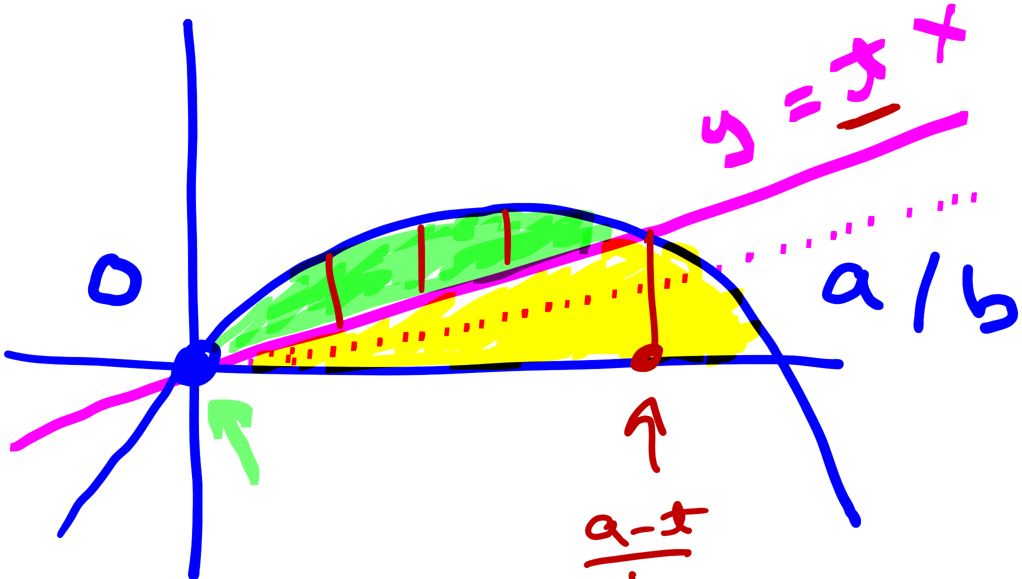
$$\int_0^{a/b} ax - bx^2\,dx = \frac{a}{2}x^2 - \frac{b}{3}x^3 \Big|_0^{a/b}$$

$$= \frac{a^3}{2b^2} - \frac{a^3}{3b^2}$$

$$= \frac{3a^3 - 2a^3}{6b^2} = \frac{a^3}{6b^2}$$

x

HW 6, Prob 6, Part B:
What is the slope of the line $y = tx$ that cuts this region into two equal area peices?



Compute area between para. & line (green)

set $= \frac{1}{2}$ total area

$$= \frac{1}{2} \cdot \frac{a^3}{6b^2}$$

$$ax - bx^2 = tx$$

$$ax - tx = bx^2$$

$$a - t = bx$$

$$\frac{a-t}{b} = x$$

$$\int_0^{\frac{a-\tau}{b}} \left[ax - bx^2 - \tau x \right] dx$$

$$= \int_0^{\frac{a-\tau}{b}} (a-\tau)x - b\,x^2\,dx$$

$$= \frac{(a-\tau)}{2}x^2 - \frac{b}{3}x^3 \Big|_0^{a-\tau/b}$$

$$= \frac{(a-\tau)}{2}\left(\frac{a-\tau}{b}\right)^2 - \frac{b}{3}\left(\frac{a-\tau}{b}\right)^3$$

$$= \frac{(a-\tau)^3}{2b^2} - \frac{(a-\tau)^3}{3b^2}$$

$$= \frac{3(a-\tau)^3 - 2(a-\tau)^3}{6b^2}$$

$$= \frac{(a-\tau)^3}{6b^2}$$ = Area between curves

$$\frac{(a-x)^3}{\cancel{6}\,\cancel{b^2}} = \frac{1}{2} \cdot \frac{a^3}{\cancel{6}\,\cancel{b^2}}$$

$$(a-x) = 2^{-1/3}\,a$$

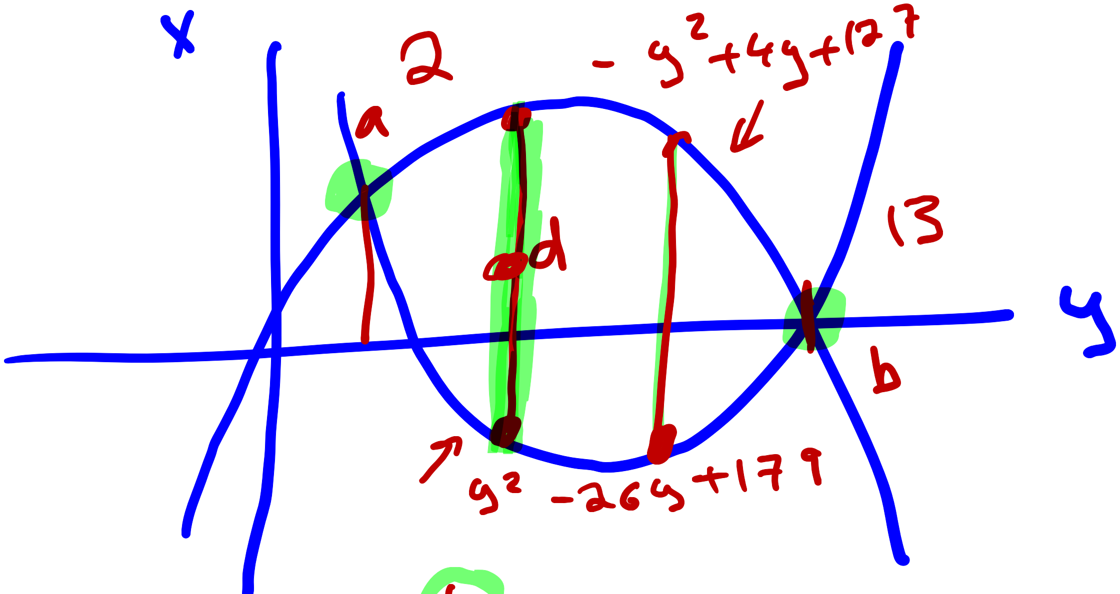
$$\boxed{x = a - a2^{-1/3}}$$

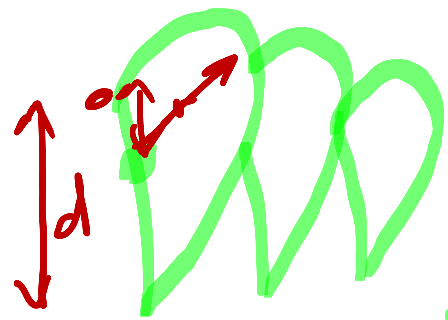
$$a(1-2^{-1/3})$$

$a=2 \quad b=7$

$$x = 2(1-2^{-1/3})$$

HW 6, Prob 11:
The base of a volume in the xy-plane is bounded by $x = -y^2 + 4y + 127$ and $x = y^2 - 26y + 179$. Every cross section perpendicular to the y-axis is a semi-circle. Find the volume.





$$\text{Area} = \frac{\pi}{2} r^2 = \frac{\pi}{2}\left(\frac{d}{2}\right)^2 = \frac{\pi}{8} d^2$$

$$\int_a^b \frac{\pi}{8} d^2 \, dy$$

$$d = f(x) - g(x)$$

$$d^2 = \left(\frac{f-g}{2}\right)^2$$

$$-y^2 + 4y + 127 = y^2 - 26y + 179$$

$$0 = 2y^2 - 30y + 52$$

$$0 = 2(y^2 - 15y + 26)$$

$$= 2(y-2)(y-13)$$

$$y = 2 \qquad y = 13$$

$$\frac{\pi}{8}\int_2^{13} d^2\, dy$$

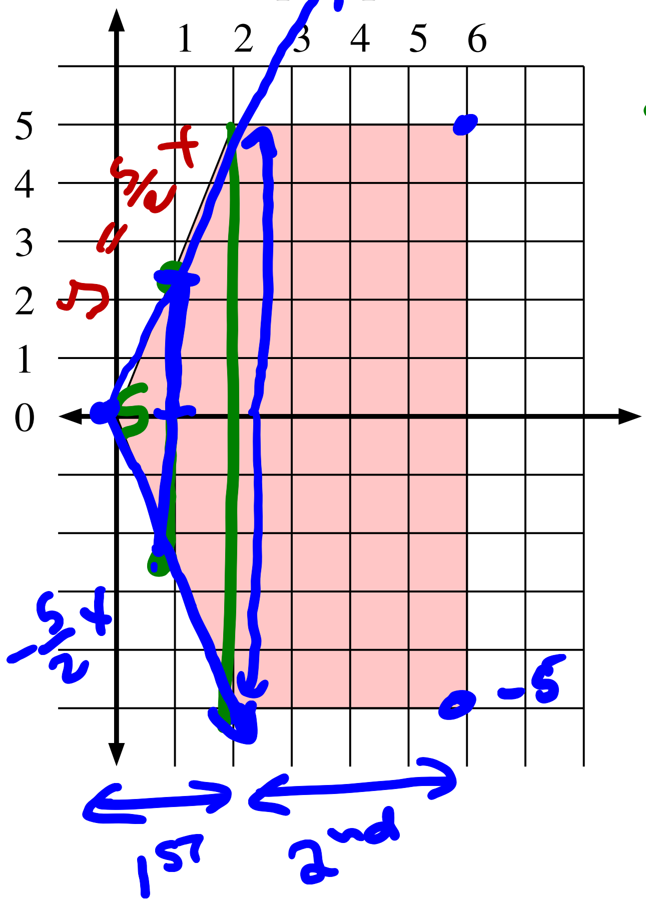
$$= \frac{\pi}{8}\int_2^{13} \left(-2y^2 + 30y - 52\right)^2 dy$$

$$= \frac{\pi}{8}\int_2^{13} y^4 - 30y^3 + 277y^2 + 780y + 676 \; dy$$

calc. $$= \frac{\pi}{8}\left[\frac{1}{5}y^5 - \frac{30}{4}y^4 + \frac{277}{3}y^3 - \frac{780}{2}y^2 + 676y\right]_2^{13}$$

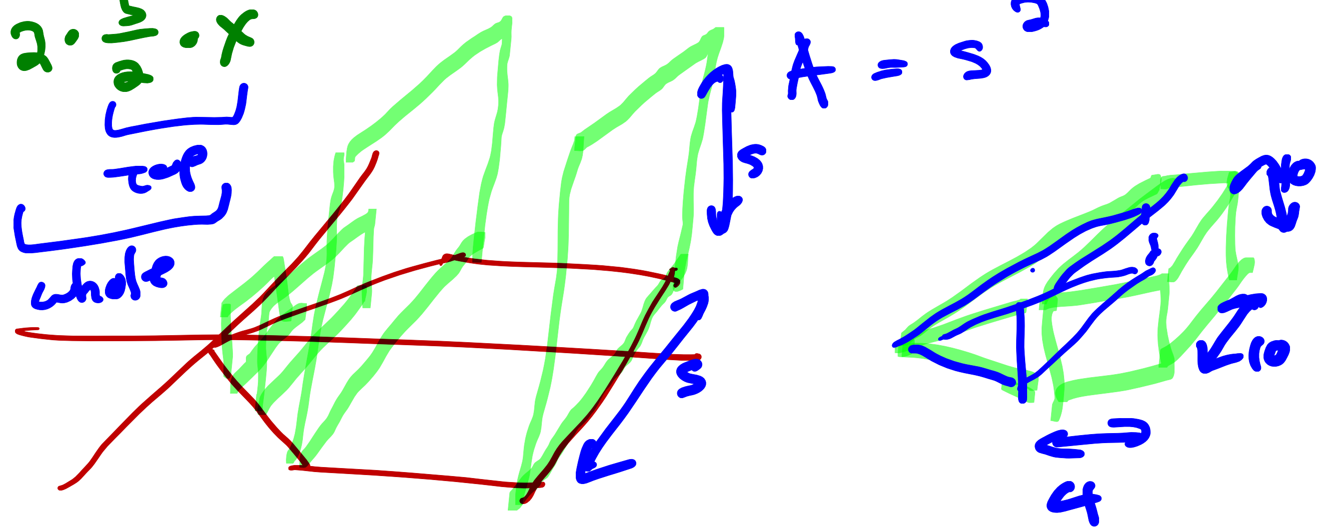
$$= \frac{\pi \cdot 161051}{60}$$

HW 6, Prob 11: The base of a volume is shown. Suppose the cross sections perpendicular to the x-axis are squares. What is the volume?



$s = 2 \cdot \underbrace{\underbrace{\tfrac{5}{2} \cdot x}_{\text{top}}}_{\text{whole}}$

$A = s^2$



2^{nd} = easy $= 400$ $\quad 4 \times 10 \times 10$

$$1^{st} = \int_0^2 \left(2 \cdot \frac{5}{2} x\right)^2 dx = \int_0^2 25 x^2 \, dx$$

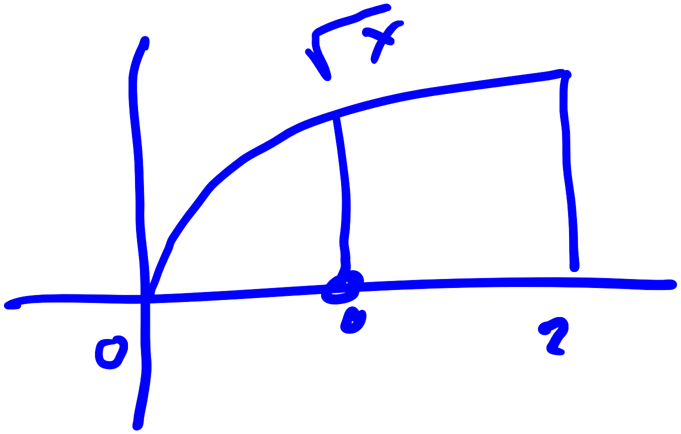
$$= \frac{25}{3} x^3 \Big|_0^2$$

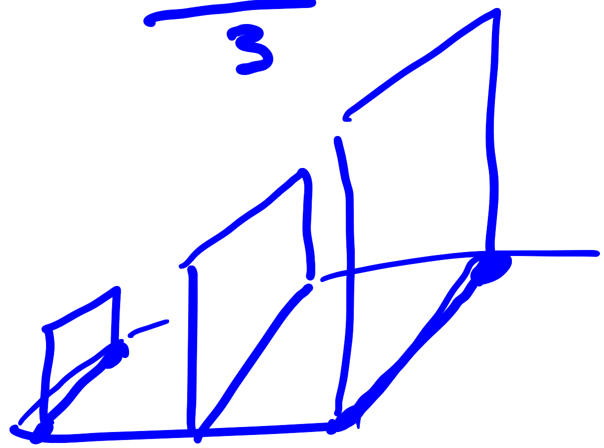
$$= \frac{25}{3} \cdot 8 - 0$$

line: $y = \frac{5}{2} x$

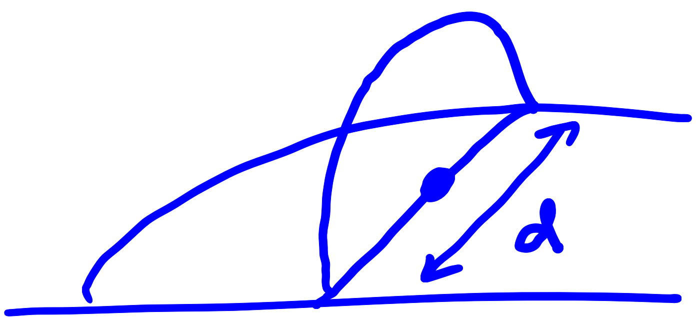
$$\text{Total vol} = \frac{200}{3} + 400$$

$$= \frac{1400}{3}$$

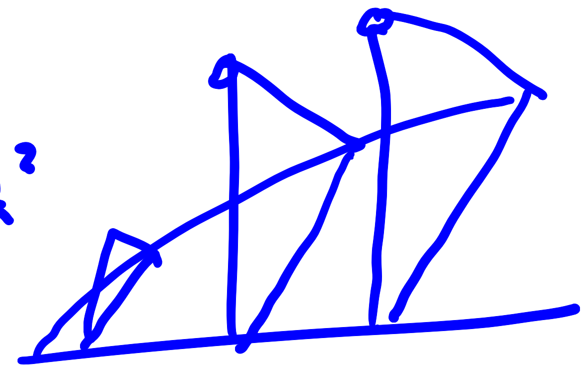




$$\int_0^2 (\sqrt{x})^2\,dx$$



$$A = \frac{\pi}{8} d^2$$

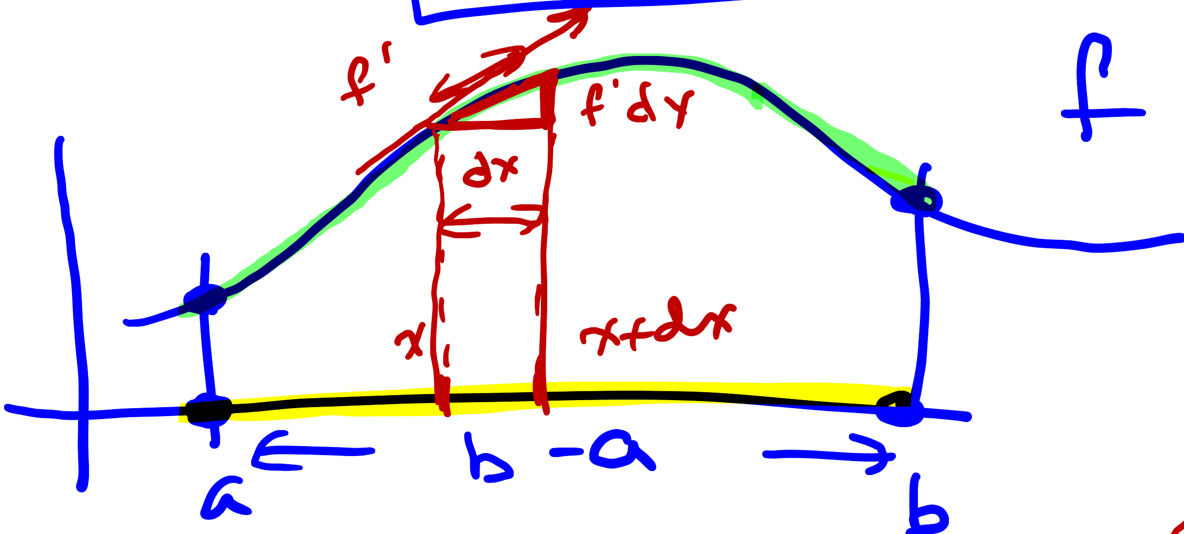


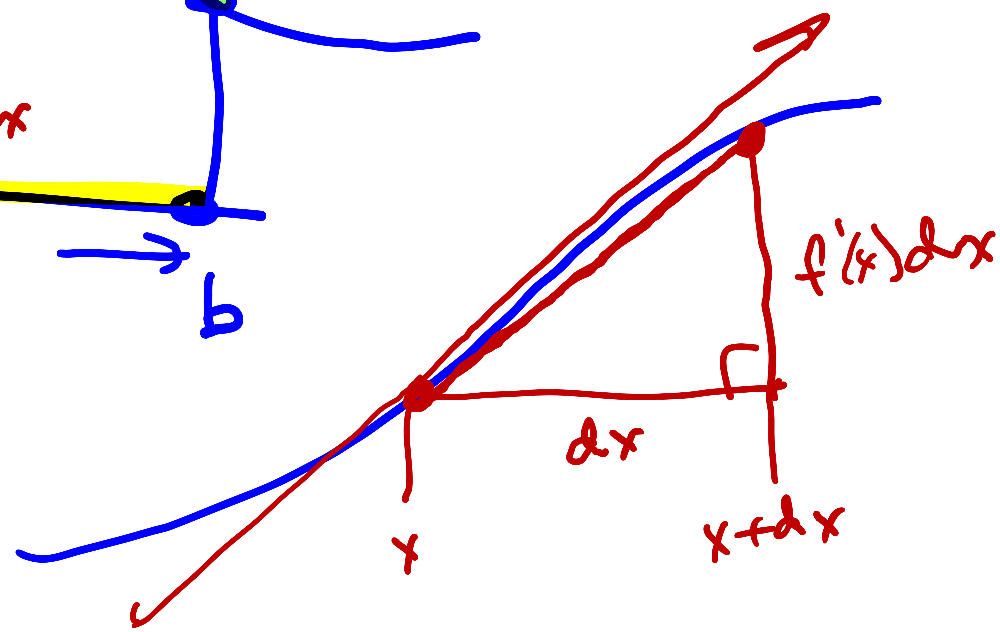
Arclength: The arclength of the graph of f over $[a, b]$ is

Quiz

$$\boxed{\int_a^b \sqrt{1 + |f'(x)|^2} dx}$$

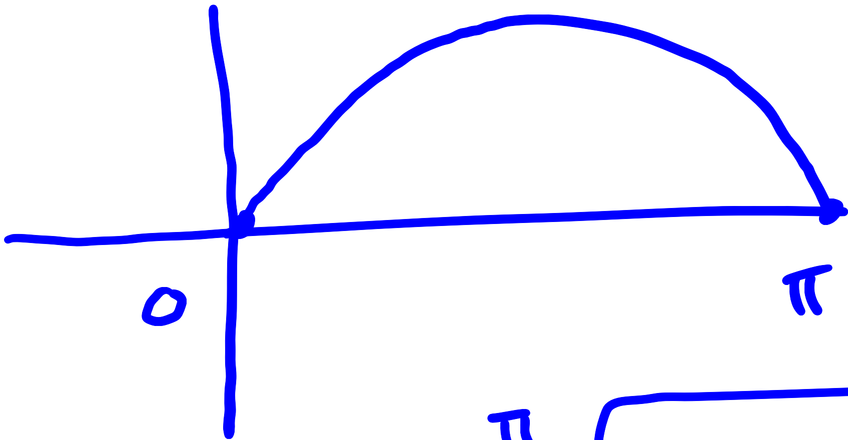
$c = \sqrt{a^2 + b^2}$





$$\text{hyp} = \sqrt{dx^2 + |f'(x)|^2 dx^2}$$
$$= \sqrt{1 + |f'|^2} \cdot dx$$

What is the arclength of graph of $\sin(x)$ between 0 and π?



$$\int_0^\pi \sqrt{1 + |f'(x)|^2}\, dx$$

$$\int_0^\pi \sqrt{1 + \cos^2(x)}\, dx$$

✓

If f on $[a, b]$ is rotated around the x-axis, the surface area is

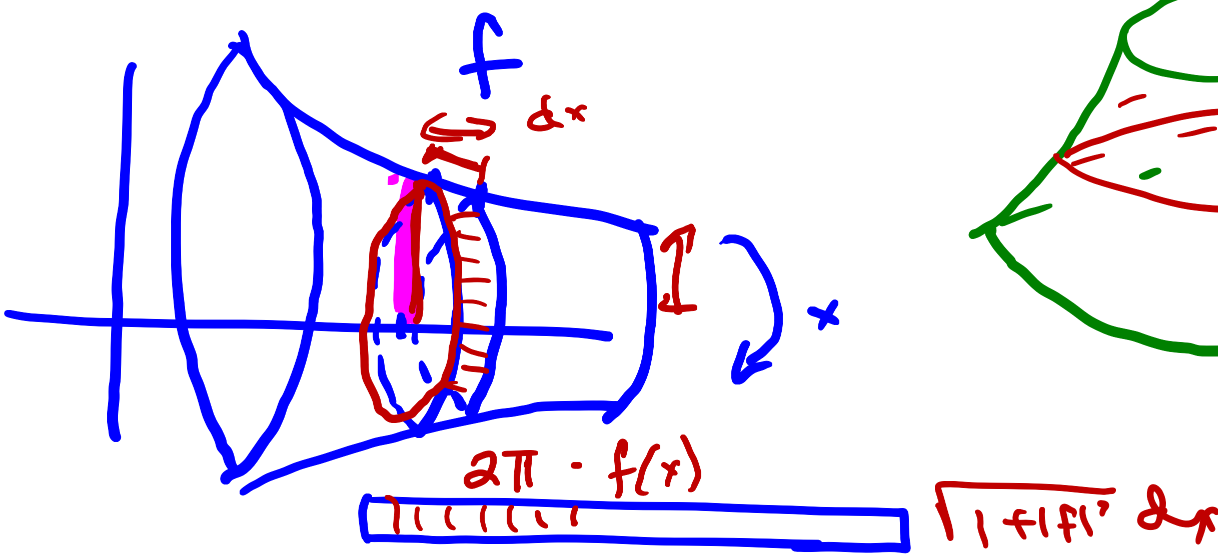
$$2\pi \int_a^b |f(x)| \sqrt{1 + |f'(x)|^2} dx.$$

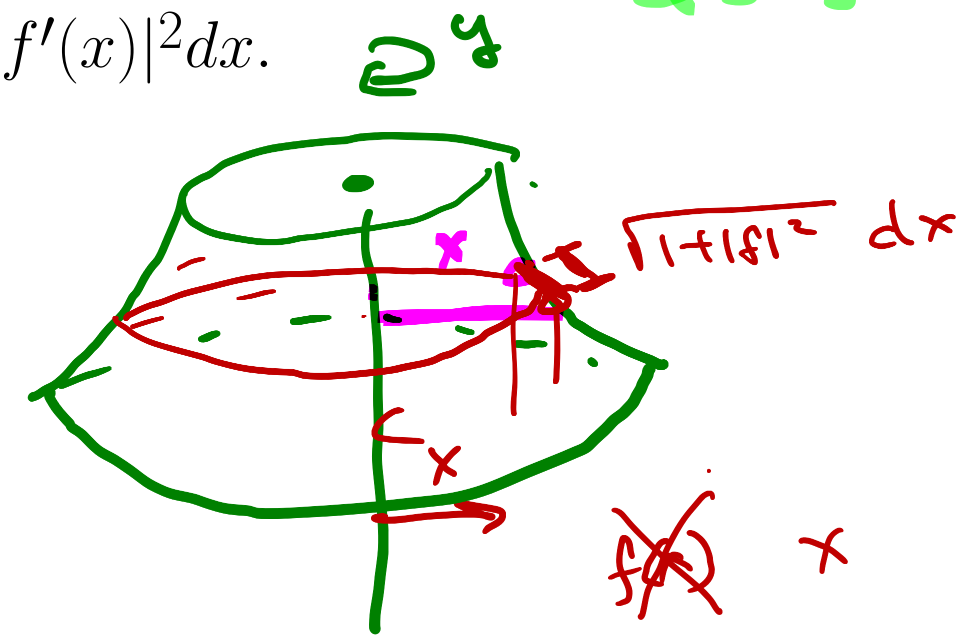
Quiz

If f on $[a, b]$ is rotated around the y-axis. The surface area is

$$2\pi \int_a^b x \sqrt{1 + |f'(x)|^2} dx.$$

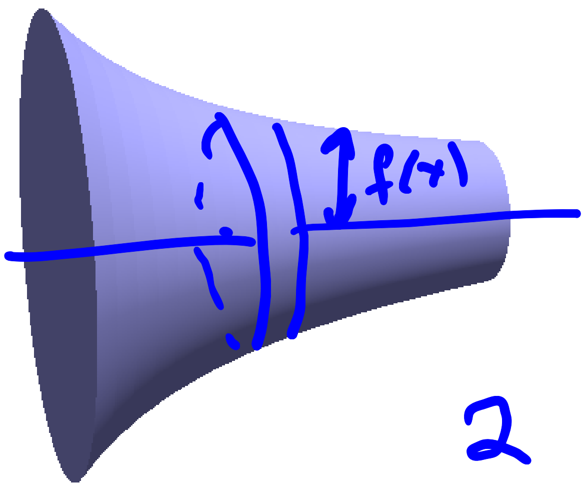
Quiz





$\frac{1}{x}$

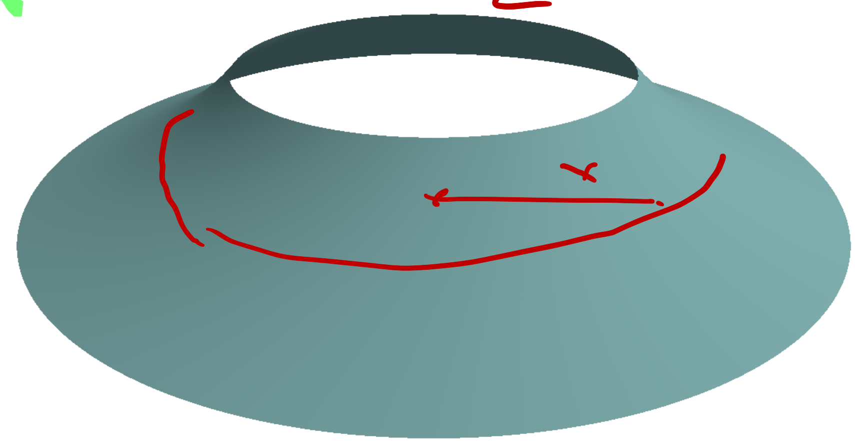
x-rot



2

1

y-rot



$$\int_1^2 2\pi x \sqrt{1+x^{-4}}\,dx$$

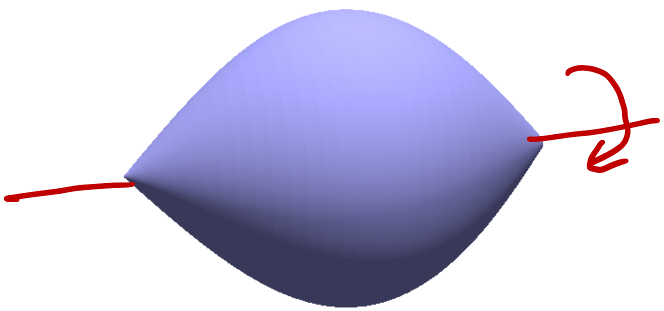
$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$\int_1^2 2\pi \cdot \frac{1}{x} \cdot \sqrt{1+\left(\frac{1}{x^2}\right)^2}\,dx$$

$$= \int_1^2 \frac{1}{x}\sqrt{1+x^{-4}}\,dx$$

What is surface area when $\sin(x)$ on $[0, \pi]$ is rotated around the x-axis?



$$\boxed{\int_0^{\pi} 2\pi \underbrace{\sin(x)}_{f(x)} \sqrt{1+\cos^2 x}\; dx}$$

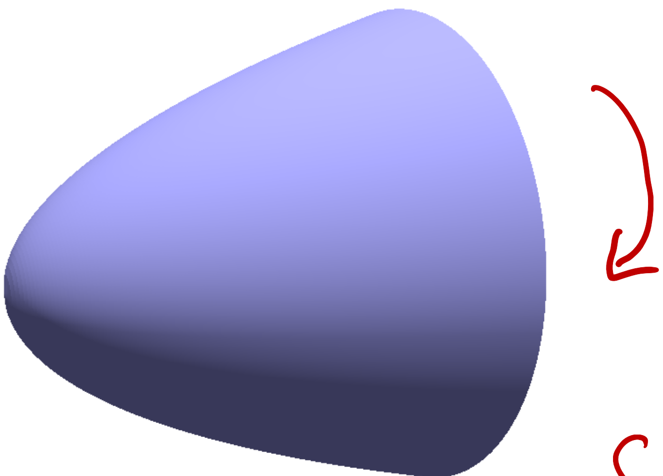
$=$

$u = \cancel{1+\cos^2}$

$du = \cancel{2\cos x}\ \sin x$

Can do this in Chap 3.

What is surface area when $\sqrt{x}$ on $[0, 1]$ is rotated around the x-axis?



$$\int 2\pi f(x)\sqrt{1+|f'|^2}\,dx$$

$$f = \sqrt{x}$$

$$f' = \frac{1}{2}\frac{1}{\sqrt{x}} = \frac{1}{2}x^{-1/2}$$

$$\int_0^1 2\pi\sqrt{x}\sqrt{1+\frac{1}{4}\frac{1}{x}}\,dx$$

$$\boxed{= 2\pi\int_0^1\sqrt{x+1/4}\,dx}$$

$u = x+1/4$

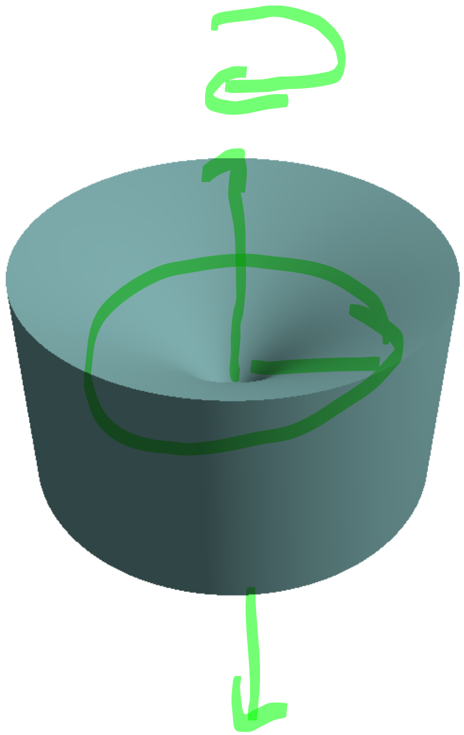
$du = dx$

$$= 2\pi\int\sqrt{u}\,du = 2\pi\frac{2}{3}u^{3/2}$$

$$= \frac{4\pi}{3}\left(1+\frac{1}{4}\right)^{3/2}\Big|_0^1 = \frac{4\pi}{3}\left(\left(\frac{5}{4}\right)^{3/2} - \left(\frac{1}{4}\right)^{3/2}\right)$$

$$= \frac{4\pi}{3}\frac{1}{8}\left(5^{3/2} - 1^{3/2}\right) = \boxed{\frac{\pi}{6}\left(5^{3/2}-1\right)}$$

What is surface area when $\sqrt{x}$ on $[0, 1]$ is rotated around the y-axis?



$$\int_0^1 2\pi \, x \sqrt{1 + |f'|^2} \, dx$$

$$= 2\pi \int_0^1 x \sqrt{1 + \frac{1}{4x}} \, dx$$

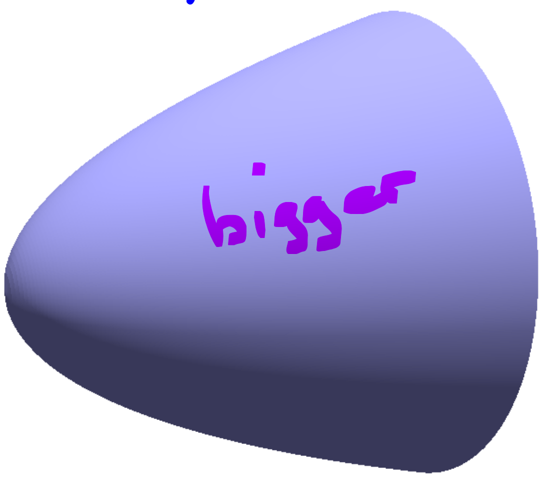
$$= \boxed{2\pi \int_0^1 \sqrt{x^2 + x/4} \, dx}$$

$$2\sqrt{3} = \sqrt{12}$$

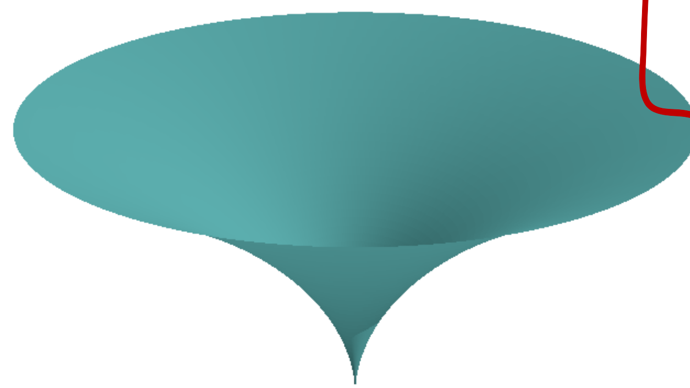
$$= \sqrt{4 \cdot 3}$$

Which is larger: the area of $\sqrt{x}$ on $[0, 1]$ rotated around the x-axis or the y-axis?

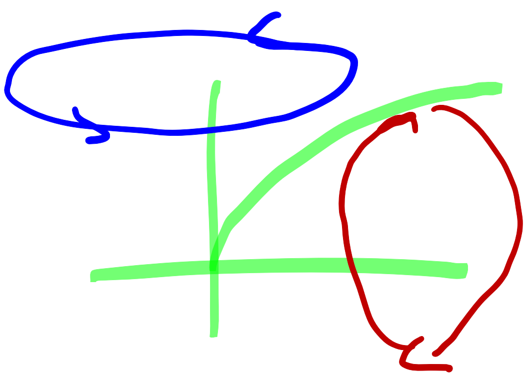
x-rot



y-rot.



Similar ques on quiz 6

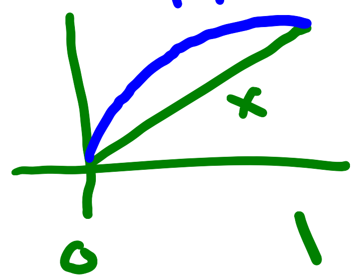


bigger

$$\boxed{2\pi\int_0^1 \sqrt{x}\,\sqrt{1+1/4x}\;dx}$$

$$2\pi\int_0^1 x\,\sqrt{1+1/4x}\;dx$$

same



$\sqrt{x} > x$

Thursday

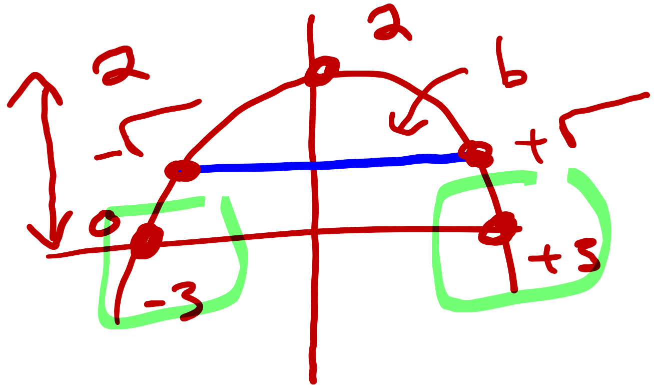
1. word problem
physical app] 45

2. Review Quiz 6] 30

OFFICE HOURS

HW 6, Q 10

$$y = 2 - \frac{2}{9}x^2, \quad y = 0$$

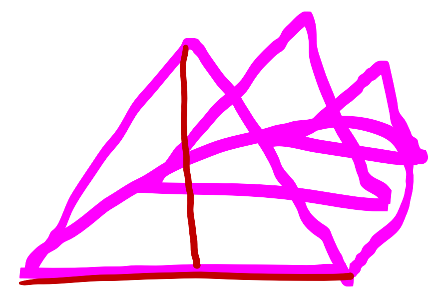


$$2 - \frac{2}{9}x^2 = 0$$

$$2 = \frac{2}{9}x^2$$

$$9 = x^2$$

$$\pm 3 = x$$



base = height

$$\text{Area} = \frac{1}{2} b \cdot h$$

$$= \underbrace{\frac{1}{2} b^2}$$

$$y = 2 - \frac{2}{9}x^2$$

$$\frac{2}{9}x^2 = 2 - y$$

$$x^2 = \frac{9}{2}(2-y)$$

$$x = \pm\sqrt{\frac{9}{2}(2-y)}$$

$$b = 2\sqrt{\frac{9}{2}(2-y)}$$

$$b^2 = 4\,\frac{9}{2}(2-y)$$

$$= 18(2-y)$$

$$\int_0^2 \frac{1}{2} b^2 \, dy$$

$$= \boxed{\int_0^2 \frac{1}{2} \left(18(2-y)\right) dy}$$

1st

$$= 9 \int_0^2 2-y \, dy$$

$$= 9 \left[2y - \frac{1}{2} y^2 \right]_0^2$$

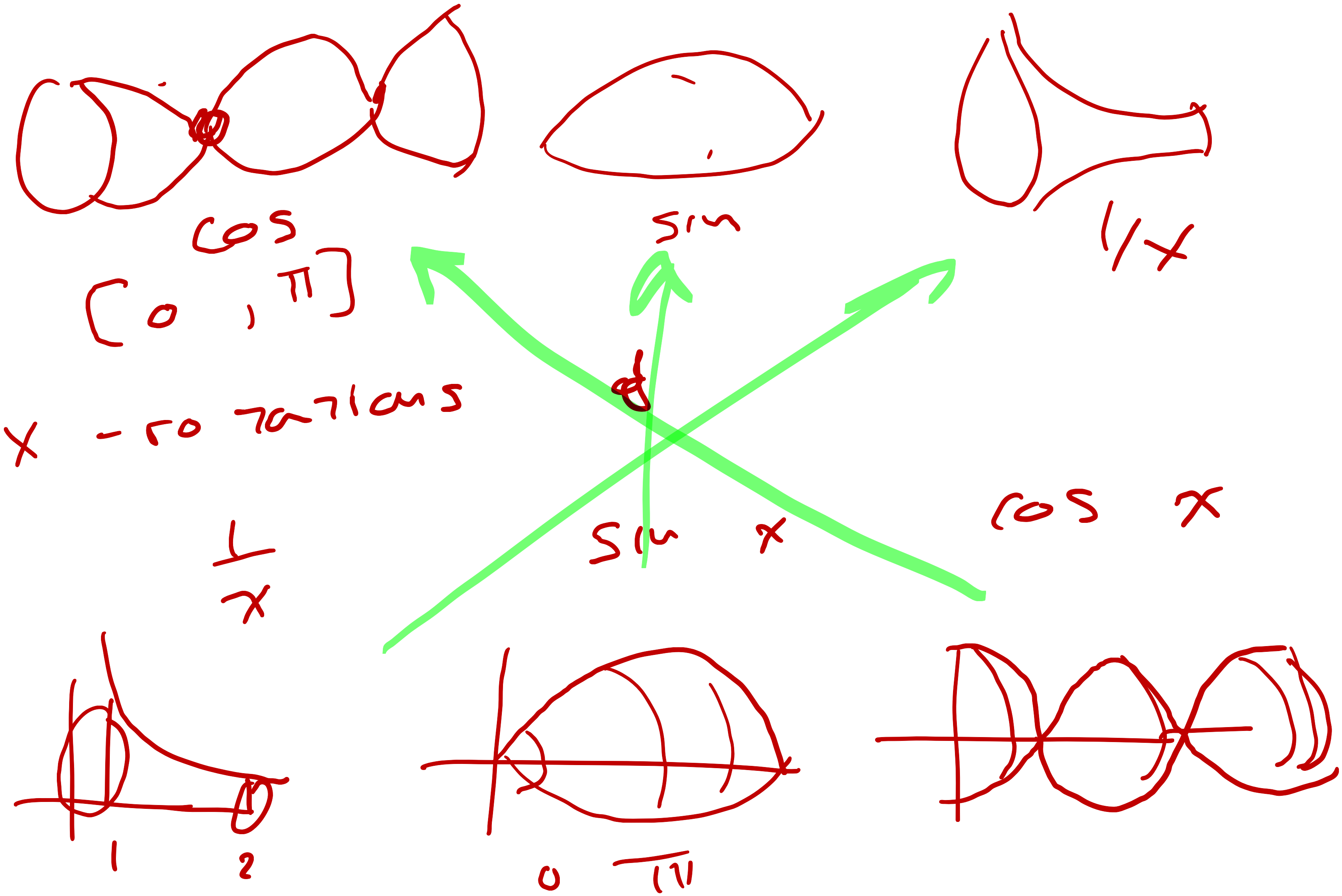
$$= 9 \left[4 - \frac{1}{2} 4 - 0 - 0 \right]$$

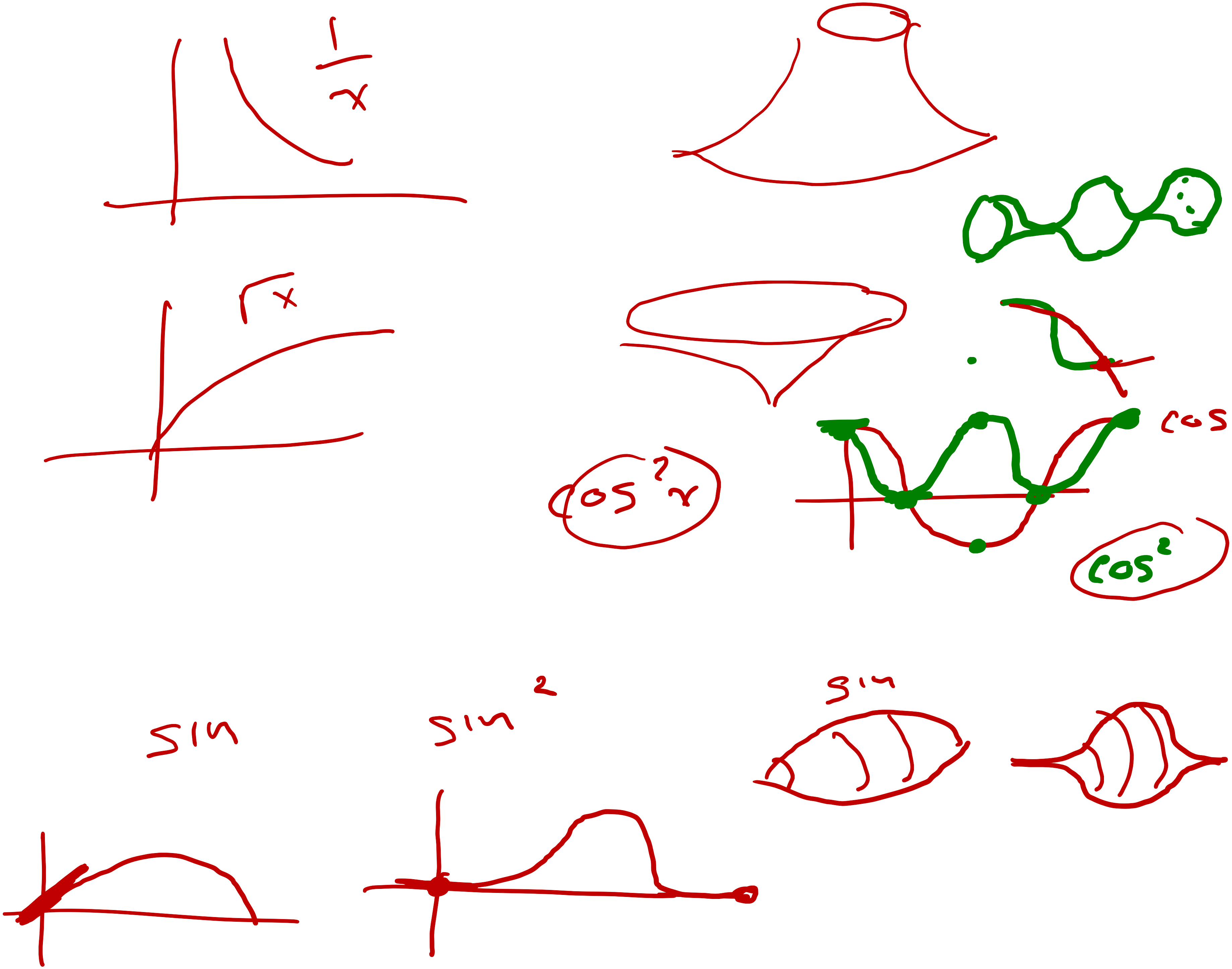
$$= 9 \left[4 - 2 \right]$$

$$\boxed{= 18}$$

2nd

On Quiz 5 (this week)



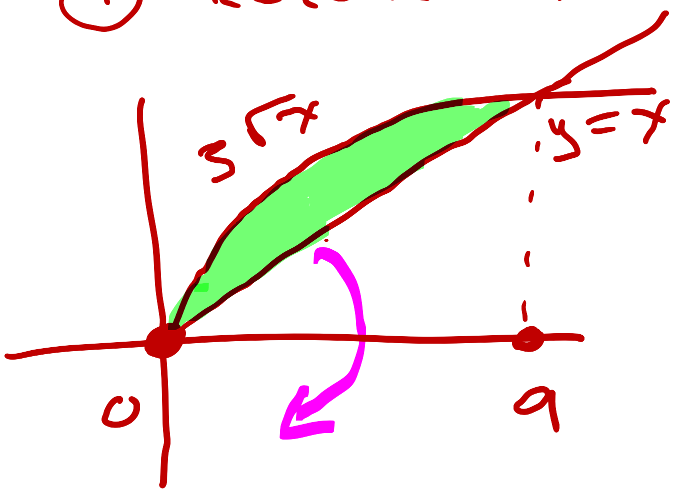


1/x
√x
cos²x
cos
cos²
sin
sin²
sin

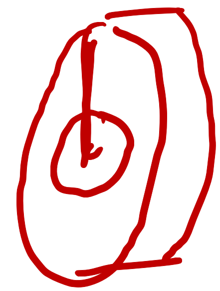
HW 6, Q. 17

R bounded by $f(x)=3\sqrt{x}$, $f(x)=x$

(1) Rotate x-axis (2) y-axis.



$$3\sqrt{x} = x$$
$$3 = \sqrt{x}$$
$$9 = x$$



$$\int_0^9 \pi\left[f^2 - g^2\right]dx = \pi\int_0^9 (3\sqrt{x})^2 - (x)^2$$
$$= \pi\int_0^9 9x - x^2\,dx$$
$$= \pi\left[\frac{9}{2}x^2 - \frac{1}{3}x^3\right]_0^9$$
$$= \pi\left[\frac{9^3}{2} - \frac{9^3}{3} - 0\right]$$

$\int f^2 - g^2$ ✓

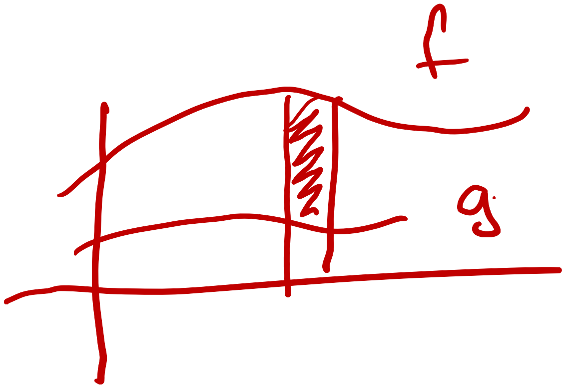
$\cancel{\int (f-g)^2}$

$= \frac{\pi}{6} \cdot 9^3$

$= \frac{729\pi}{6}$

$= \frac{243\pi}{2}$

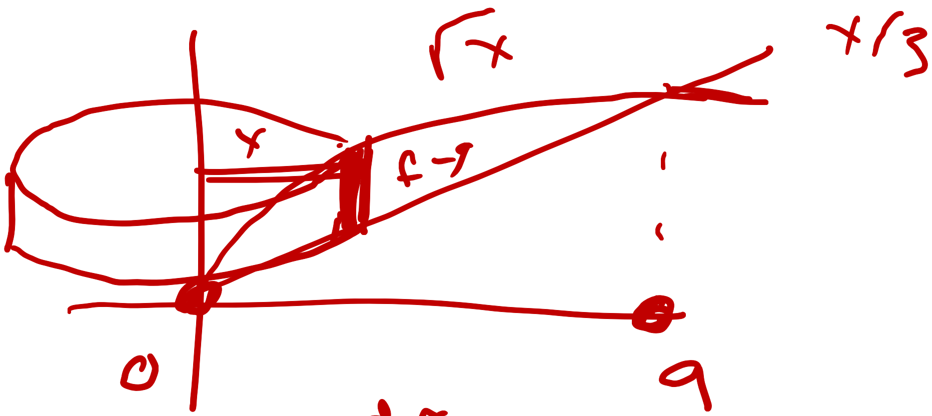
$$\begin{array}{r} 81 \\ \times\ 9 \\ \hline 729 \end{array}$$



$\boxed{\pi f(x)^2} - \boxed{\pi g(x)^2}$

HW 6, # 17 Part b.

$f(x) = \sqrt{x}$ $\qquad g(x) = \dfrac{x}{3}$

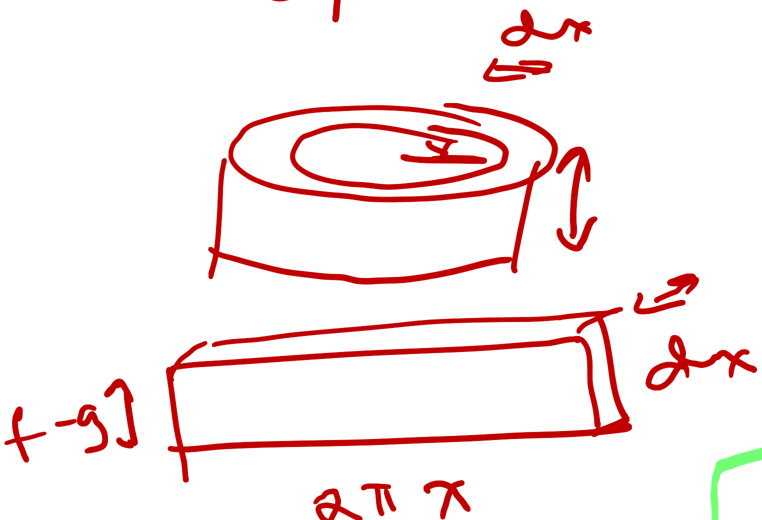


$$\sqrt{x} = \frac{x}{3}$$

$$3\sqrt{x} = x$$

$$3 = \sqrt{x}$$

$$9 = x$$



$$\int_0^9 2\pi x \cdot [f(x) - g(x)]\, dx$$

$$= \boxed{\int_0^9 2\pi \underline{x} \left[\sqrt{x} - \underline{\frac{x}{3}}\right] dx}$$

$$= \boxed{2\pi \int_0^9 x^{3/2} - \frac{1}{3} x^2 \, dx}$$

$$= 2\pi \left[\frac{2}{5} x^{5/2} - \frac{1}{3} \cdot \frac{1}{3} x^3 \right]_0^9$$

$$= 2\pi \left[\frac{2}{5} 9^{5/2} - \frac{1}{9} 9^3 - 0 - 0 \right]$$

$$= 2\pi \left[\frac{2}{5} 9^{5/2} - 9^2 \right] \checkmark$$

$$9^2 \cdot \sqrt{9}$$

$$= 2\pi \, 9^2 \left[\frac{2}{5} \sqrt{9} - 1 \right]$$

$$= 2\pi \, 9^2 \left[\frac{6}{5} - 1 \right]$$

$$= 2\pi \, 9^2 \, \frac{1}{5} = \frac{\pi \cdot 2 \cdot 81}{5}$$

$$= \boxed{\frac{162}{5} \pi}$$