MAT 126.01, Prof. Bishop, Thursday, Oct 29, 2020

Section 3.2: Trigonometric Integrals

Section 3.3: Trigonometric Substitution

Quiz 8 review

1. May use page of notes on Quiz 8 next week.

2. Tuesday Nov 3 - lecture pre-recorded, no office hours

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.
(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \cos^3 x \sin x dx$ $= -\int u^3 du$

$u = \cos x$

$du = -\sin x dx$

$= -\frac{1}{4}u^4 + C$

$= -\frac{1}{4}\cos^4 x + C$

$\cos^2 + \sin^2 = 1$

$\sin^2 = 1 - \cos^2$

$\int \cos^3 x \sin^3 x$

$\int \cos^3 x \sin^2 x \sin x$

$\int \cos^3 (1 - \cos^2) \sin x = \int [\cos^3 - \cos^5] \sin x$

$= -\int u^3 - u^5 du$

$= -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.
(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

$$\cos^2 y = \frac{1}{2}(1 + \cos 2y)$$

$$\cos^2 2x = \frac{1}{2}(1 + \cos(\cancel{2 \cdot 2x}\ 4x))$$

$$y = 2x$$

Evaluate $\int \sin^4 x dx$

use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$\int \sin^4 x \, dx = \int (\sin^2 x)^2 dx$$

$$= \int \left(\frac{1}{2}(1 - \cos 2x)\right)^2 dx$$

$$= \frac{1}{4} \int (1 - \cos 2x)^2 dx$$

$$= \frac{1}{4} \int 1 - 2\cos 2x + \cos^2 2x$$

$$= \frac{1}{4} \int 1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x \, dx$$

$$= \frac{1}{4}\left[x - \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right] + C$$

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.
(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \cos^4 x \sin^3 x dx$

$$= \int \cos^4 x \sin^2 x \cdot \sin x \, dx$$

$$= \int \cos^4 x (1 - \cos^2 x) \sin x \, dx$$

$$= \int (\cos^4 x - \cos^6 x) \sin x \, dx$$

$$u = \cos x \qquad du = -\sin x \, dx$$

$$= -\int u^4 - u^6 \, du$$

$$= \frac{1}{7} u^7 - \frac{1}{5} u^5 + C$$

$$= \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$$

To integrate products of $\sin(ax), \sin(bx), \cos(ax), \cos(bx)$ use:

(d) $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

Evaluate $\int \sin(5x)\cos(3x)dx$.

$a=5,\ b=3$

$$= \int \frac{1}{2}\sin(2x) + \frac{1}{2}\sin(8x)\,dx$$

$$= -\frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x) + C$$

$(\tan)' = \sec^2 \qquad (\sec)' = \tan \cdot \sec$

To integrate $\boxed{\int \tan^k x \sec^j x dx}$:

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.
(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$. Then use $u = \sec x$.
(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.
(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \tan^6 x \sec^4 x dx$.

$= \int \tan^6 x \boxed{\sec^2 x} \sec^2 x \, dx$

$= \int \tan^6 x \boxed{(\tan^2 x + 1)} \sec^2 x \, dx$

$= \int (\tan^8 x + \tan^6 x) \underbrace{\sec^2 dx}$

$u = \tan x \qquad du = \sec^2 x$

$= \int u^8 + u^6 \, du$

$= \frac{1}{9} u^9 + \frac{1}{7} u^7 + C \quad = \frac{1}{9} \tan^9 x + \frac{1}{7} \tan^7 x + C$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\boxed{\tan^2 + 1 = \sec^2}$

$\boxed{\tan^2 = \sec^2 - 1}$

$$(\sec x)' = \tan x \sec x$$

To integrate $\int \tan^k x \sec^j x dx$:

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.
(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$. Then use $u = \sec x$.
(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.
(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \tan^3 x \sec^3 x dx$.

$$= \int \tan^2 x \tan x \sec^2 x \sec x \, dx$$

$$= \int \tan^2 x \sec^2 x \underbrace{(\tan x \sec x)dx}$$

$$u = \sec x \qquad du = \tan x \sec x \, dx$$

$$= \int (\sec^2 x - 1) \sec^2 x (\tan x \sec x) dx$$

$$= \int (u^2 - 1) u^2 du$$

$$= \int u^4 - u^2 du$$

$$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C$$

$$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

Verify $\int \sec x dx = \ln(\sec x + \tan x) + C$

$$\Big(\qquad \Big)' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x \left(\cancel{\tan x + \sec x} \right)}{\cancel{\tan x + \sec x}}$$

Evaluate $\boxed{\int \sec^3 x\,dx.}$

"integrate by parts"

$$= \int \underbrace{\sec^2 x}_{(\tan)'} \cdot \sec x\,dx$$

$dv = \sec^2 x\,dx \qquad u = \sec x$

$v = \tan x \qquad du = \sec x \tan x$

$$= uv - \int v\,du$$

$$= \sec x \tan x - \int \tan x \sec x \tan x\,dx$$

$$= \sec x \tan x - \int \tan^2 x \sec x\,dx$$

$$= \sec x \tan x - \int (\sec^2 x - 1)\sec x\,dx$$

$$= \sec x \tan x \boxed{- \int \sec^3 x} + \int \sec x\,dx$$

$$\cancel{2} \int \sec^3 x\,dx = \frac{\sec x \tan x + \ln(\sec x + \tan x)}{2}$$

Not on Quiz 8

Section 3.3: Integration involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$.

Some such integrals we can already do:

$$\int \frac{dx}{\sqrt{a^2-x^2}}, \qquad \int \frac{x\,dx}{\sqrt{a^2-x^2}}, \qquad \int x\sqrt{a^2-x^2}\,dx$$

Integrals involving $\sqrt{a^2 - x^2}$.

Use substitution $x = a\sin\theta$, $dx = a\cos\theta d\theta$.
Converts $\sqrt{a^2 - x^2}$ to $a\cos\theta$.
Simplify.
Evaluate.
Use reference triangle to convert $\theta = \sin^{-1}(x/a)$ back to x.

$$\boxed{\sqrt{a^2 - x^2}} \rightarrow \sqrt{a^2 - (a\sin\theta)^2}$$

$$\sqrt{a^2 - a^2\sin^2\theta}$$

$$a\sqrt{1 - \sin^2\theta}$$

$$a\sqrt{\cos^2\theta}$$

$$\boxed{a\cos\theta}$$

$\sin^2 + \cos^2 = 1$

$\cos^2 = 1 - \sin^2$

Integrals involving $\sqrt{a^2 - x^2}$, use substitution $x = a\sin\theta$.

Evaluate $\int\sqrt{9 - x^2}\,dx$

$$\int \sqrt{9-x^2}\,dx = \int \sqrt{9-9\sin^2\theta}\;3\cos\theta\,d\theta$$

$a = 3$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\frac{x}{3} = \sin\theta$

$\sin^{-1}\left(\frac{x}{3}\right) = \theta$

$$= \int (3\cos\theta)(3\cos\theta)\,d\theta$$

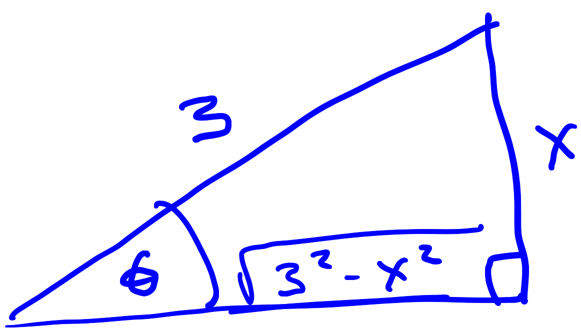
$$= 9\int \cos^2\theta\,d\theta$$

$$= 9\int \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{4}\,2\sin\theta\cos\theta + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2}\sin\left(\sin^{-1}\left(\frac{x}{3}\right)\right) \cdot \cos\left(\sin^{-1}\frac{x}{3}\right)$$



$\sin\theta = \frac{x}{3}$

$\frac{\text{opp}}{\text{hyp}}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2}\,\frac{x}{3}\cdot$$

$$\cos\left(\sin^{-1}\frac{x}{3}\right) = \frac{\sqrt{9-x^2}}{3}$$

Integrals involving $\sqrt{a^2-x^2}$, use substitution $x = a\sin\theta$.

Evaluate $\int \frac{1}{x}\sqrt{4-x^2}\,dx$.

$$= \int \frac{1}{2\sin\theta}\cdot\sqrt{4-4\sin^2}\cdot 2\cos\theta\, d\theta$$

$a=2$

$$x = 2\sin\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$= \int \frac{2\cos\theta \cdot \cancel{2}\cos\theta}{\cancel{2}\sin\theta}\, d\theta$$

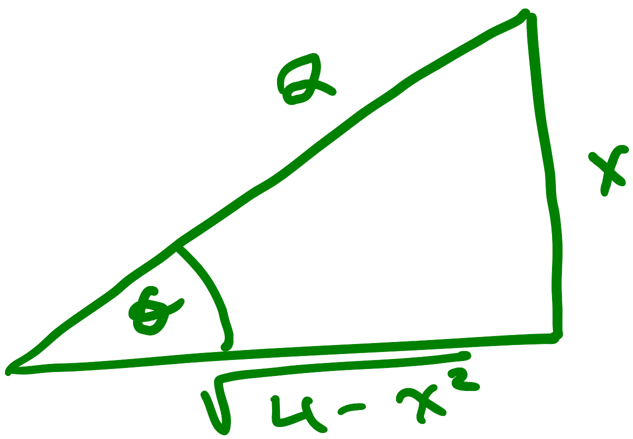
$$= 2\int \frac{\cos^2}{\sin}\, d\theta$$

$$= 2\int \frac{(1-\sin^2)}{\sin}\, d\theta$$

$$= 2\int \csc - \sin\, d\theta$$

$$= 2\left[\ln \csc\theta - \cot\theta + \cos\theta\right] + C$$

$$= 2\left[\ln\left(\frac{2}{x} - \frac{\sqrt{4-x^2}}{x}\right) + \frac{\sqrt{4-x^2}}{2}\right] + C$$



$$\sin\theta = \frac{x}{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}} = \frac{2}{x}$$

$$\cot = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{4-x^2}}{x}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{4-x^2}}{2}$$

Integrands involving $\sqrt{a^2 + x^2}$.

Check for alternate methods.
Substitute $x = a \tan \theta$ and $dx = a \sec^2 \theta d\theta$.
Converts $\sqrt{a^2 + x^2}$ to $a \sec \theta$.
Simplify.
Evaluate trig integral.
Use reference triangle to convert back to x.

Integrals involving $\sqrt{a^2 + x^2}$, use substitution $x = a\tan\theta$.

Evaluate $\int \frac{dx}{\sqrt{1+x^2}}$

Integrals involving $\sqrt{a^2 + x^2}$, use substitution $x = a \tan \theta$.

Find the arclength of $y = x^2$ over $[0, 1]$.

Integrands involving $\sqrt{x^2 - a^2}$.

Check for alternate methods.
Use $x = a \sec \theta$ and $dx = a \sec \theta \tan \theta d\theta$.
Simplify.
Evaluate.
Use reference triangle to convert to x.

Integrals involving $\sqrt{x^2 - a^2}$, use substitution $x = a \sec \theta$.

Evaluate $\int_3^5 \sqrt{x^2 - 9}dx$

Quiz 8 review: Only Sections 3.1 and 3.2.

Page 1: Integration by parts.

- 2 simple integration by parts
- 2-part problem: use $dv = 1$, integrate by parts twice and solve for integral
- integrate by parts using $dv = 1$ and $\ln x$.

Page 2: Trigonometric integrals.

- 3 problems: choose correct strategy from sheet.
- 2 questions: evaluate 2 trig power integrals

Evaluate $\int x \cos x dx$

$u = x \qquad dv = \cos x\, dx$

$du = dx \qquad v = \sin x$

$= uv - \int v\, du$

$= x \sin x - \int \sin x\, dx$

$= x \sin x + \cos x + C$

$(\quad)' = \cancel{\sin x} + x \cos x - \cancel{\sin x} + 0$

(1) Apply integration by parts to $\int \underbrace{\sin x}_{u}\, \underbrace{e^x dx}_{dv}$.

$$= uv - \int v\,du = \boxed{\sin x \cdot e^x - \int \underbrace{e^x}_{dv} \underbrace{\cos x}_{u}\, dx}$$

(2) Apply integration by parts again and solve for the integral above.

$$= \sin x e^x - \left[uv - \int v\,du \right]$$

$$= \sin e^x - \left[\cos x e^x - \int e^x (-\sin x)\, dx \right]$$

$$= \underline{\sin x e^x - \cos x e^x} - \int \sin x e^x dx$$

$$\frac{\cancel{2} \int \sin x e^x}{\cancel{2}} = \boxed{\frac{\sin x e^x - \cos x e^x}{2}}$$

Evaluate $\int \ln \sqrt{x} dx$.

$$\ln x^{p} = p \ln x$$

$$\ln \sqrt{x} = \ln x^{1/2}$$

$$= \frac{1}{2} \ln x$$

$$= \int \ln x^{1/2} dx$$

$$= \int \frac{1}{2} \ln x \, dx$$

$$= \frac{1}{2} \int \underbrace{1}_{dv} \cdot \underbrace{\ln x}_{u} \, dx \qquad du = \frac{1}{x} dx$$

$$= \frac{1}{2} \left[uv - \int v \, du \right]$$

$$= \frac{1}{2} \left[x \cdot \ln x - \int \underbrace{x \cdot \frac{1}{x}}_{1} dx \right]$$

$$\boxed{= \frac{1}{2} \left[x \ln x - x \right]}$$

$\boxed{e}$ What is the correct strategy to evaluate: $\int \sin(9x)\cos(8x)dx$?

(a) Replace $\sin^2 x$ by $1-\cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1-\sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ or $\cos^2 x = \frac{1}{2}(1+\cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) Use $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1}\sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.

Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.

Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

$\boxed{B}$ What is the correct strategy to evaluate: $\int \cos^7 x \sin^2 x dx$?

(a) Replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) Use $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.

Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.

Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

$\boxed{G}$ What is the correct strategy to evaluate: $\int \tan^5 x \sec^2 x dx$?

(or H)

(a) Replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) Use $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.

Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.

Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \sin^5 x \cos^5 x dx$

$$= \int \sin^4 \cos^5 \underbrace{\sin dx}$$

$$= \int (1-\cos^2)^2 \cos^5 \sin dx$$

$$u = \cos x \qquad du = -\sin x\, dx$$

$$= -\int (1-u^2)^2 u^5 \, du$$

$$= -\int (1-2u^2+u^4) u^5$$

$$= -\int u^5 - 2u^7 + u^9$$

$$= -\left[\frac{1}{6}u^6 - \frac{2}{8}u^8 + \frac{1}{10}u^{10}\right] + C$$

$$= -\frac{1}{6}\cos^6 x + \frac{1}{4}\cos^8 x - \frac{1}{10}\cos^{10} + C$$

$$6 = 4 + 2$$

Evaluate $\int \tan^2 x \sec^6 x dx$

$$= \int \tan^2 x \sec^4 x \sec^2 x dx$$

$$= \int \tan^2 (1+\tan^2)^2 \sec^2 x dx$$

$$u = \tan x, \quad du = \sec^2 x dx$$

$$= \int u^2 (1+u^2)^2 du$$

$$= \int u^2 (1+2u^2+u^4) du$$

$$= \int u^2 + 2u^4 + u^6 du$$

$$= \frac{1}{3}u^3 + \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$$

$$= \frac{1}{3}\tan^3 x + \frac{2}{5}\tan^5 x + \frac{1}{7}\tan^7 x + C$$

MAT 126, Oct 29, 2020

Office Hours

start $\approx$ 11:20

Quiz 7:

Newton's law of cooling.

#9 A turkey is at 400° in an oven and is put in a 60° room. What is equation for temp. as a function of time t? in hours.

$$T = (T_0 - T_a)e^{-kt} + T_a$$

$T_0 = 400$, $T_a = 60$

$$T = \boxed{340e^{-kt} + 60}$$

#10 Suppose after 1 hour the Turkey is at 150°. What is k?

$t = 1 \quad T = 150$

$$150 = 340e^{-k\cdot 1} + 60$$

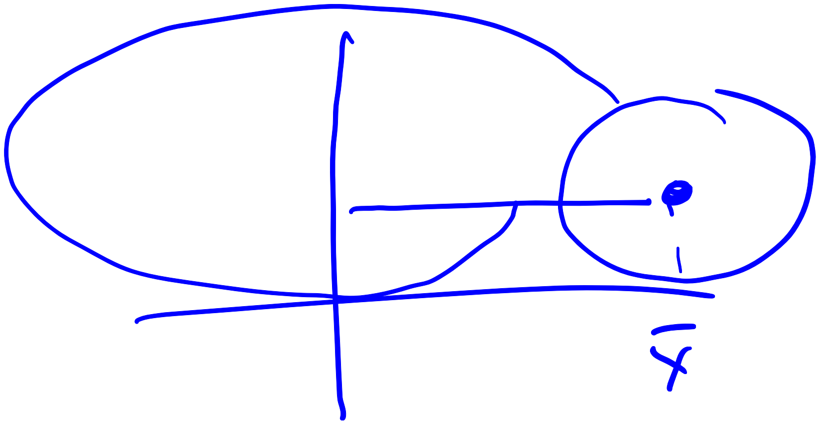
$$90 = 340e^{-k}$$

$$\frac{90}{340} = e^{-k}$$

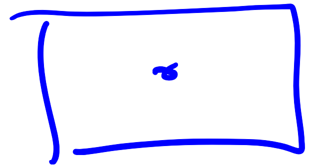
$$\ln\left(\frac{90}{340}\right) = -k$$

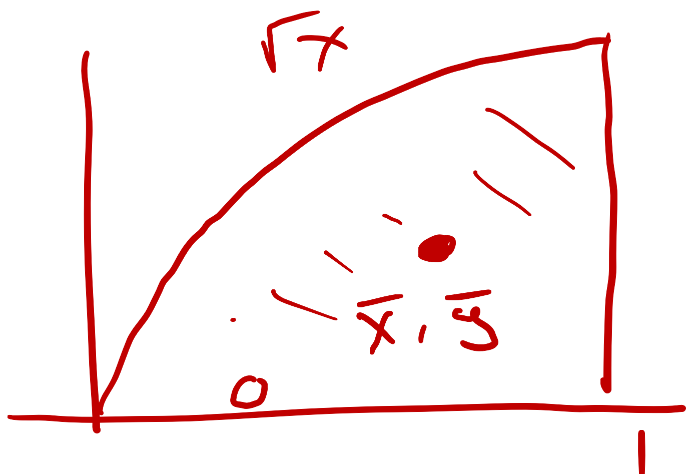
$$\ln\left(\frac{90}{340}\right)^{-1} = k$$

$$\boxed{\ln\frac{340}{90} = k}$$



$$\text{Vol} = A \cdot 2\pi \cdot \bar{x}$$





Find center of mass

$(\bar{x}, \bar{y})$ (assume density $\rho = 1$)

$$\bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}$$

$$m = \int_a^b f(x)\,dx$$

$$= \int_0^1 \sqrt{x}\,dx$$

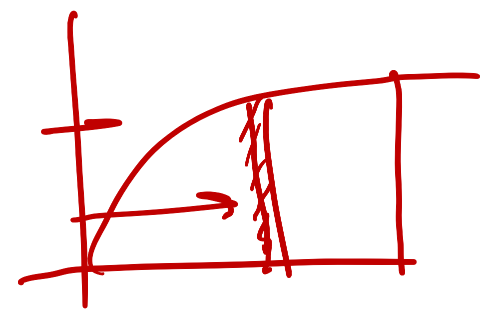
$$= \int_0^1 x^{1/2}\,dx$$

$$= \frac{2}{3} x^{3/2} \Big|_0^1$$

$$= \frac{2}{3} 1^{3/2} - \underbrace{\frac{2}{3} 0^{3/2}}_{0}$$

$$= \frac{2}{3}$$

$$M_y = \int_a^b x f(x)\,dx$$



$$= \int_0^1 x \cdot x^{1/2}\,dx$$

$$= \int_0^1 x^{3/2} = \frac{2}{5} x^{5/2} \Big|_0^1 = \frac{2}{5}$$

$$M_x = \int_a^b \frac{1}{2} f(x) \cdot f(x)\,dx$$

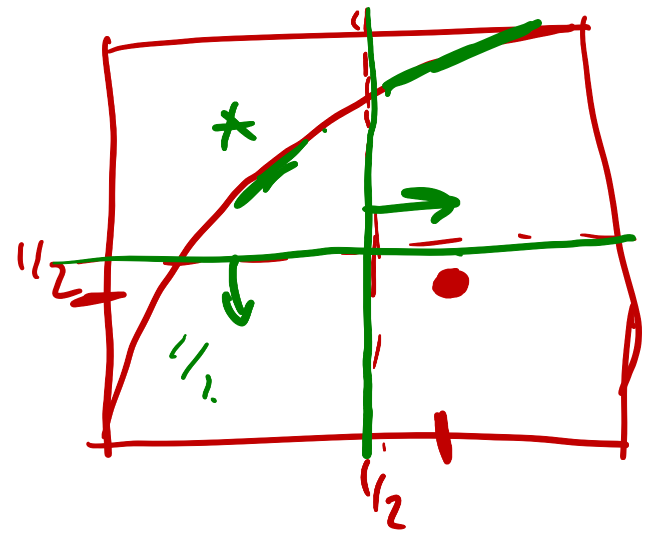
$$= \int_a^b \frac{1}{2} f(x)^2\,dx$$

$$= \int_0^1 \frac{1}{2} (\sqrt{x})^2\,dx$$

$$= \frac{1}{2} \int_0^1 x\,dx$$

$$= \frac{1}{2} \cdot \frac{1}{2} x^2 \Big|_0^1$$

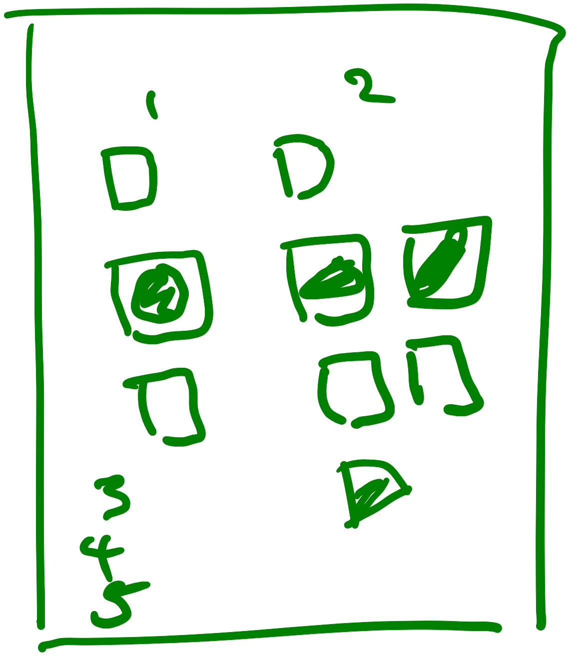
$$= 1/4$$

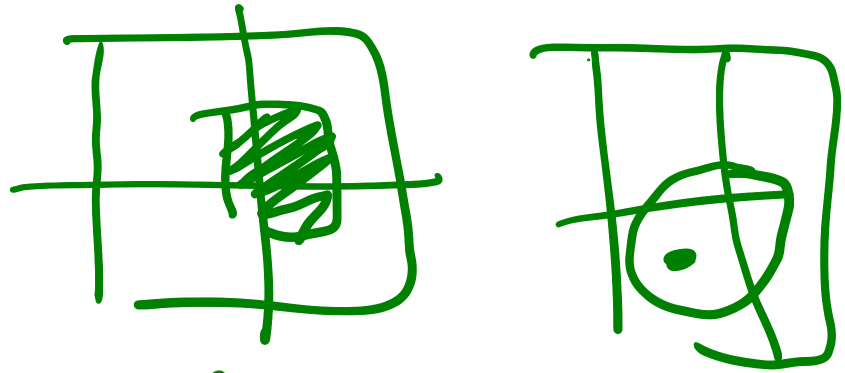


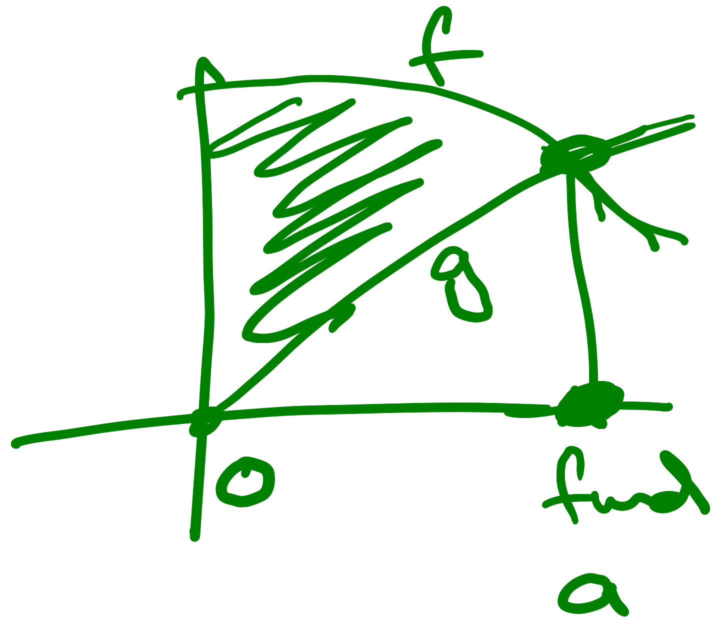
$$\bar{x} = \frac{M_y}{m} = \frac{2/5}{2/3} = 3/5$$

$$\bar{y} = \frac{M_x}{m} = \frac{1/4}{2/3} = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$$

Quiz 7







$f = g$

$m =$ fund

$M_y = \int \cdots -$

$M_x = \int \cdots$

$$\int \sin x \underbrace{e^x}_{u} \, dx = uv - \int v\, du$$

$u = e^x \qquad du = e^x \, dx$

$dv = \sin x \, dx \qquad v = -\cos x$

$$= -\cos x \, e^x - \int -\cos x \, e^x \, dx$$

$$= -\cos x \, e^x + \int e^x \cos x \, dx$$

$u = e^x \qquad du = e^x dx$

$dv = \cos x \, dx$

$v = \sin x$

$$= -\cos x \, e^x + \left[\sin x \, e^x - \int \sin x \, e^x \, dx \right]$$

$$\frac{\cancel{2} \int \sin x \, e^x \, dx}{\cancel{2}} = \frac{-\cos x \, e^x + \sin x \, e^x}{2}$$

$$= \frac{\sin x \, e^x - \cos x \, e^x}{2}$$