MAT 126.01, Prof. Bishop, Thursday, Oct 29, 2020
Section 3.2: Trigonometric Integrals
Section 3.3: Trigonometric Substitution
Quiz 8 review

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.
(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \cos^3 x \sin x dx$

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.
(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \sin^4 x dx$

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.
(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \cos^4 \sin^3 x dx$

To integrate products of $\sin(ax), \sin(bx), \cos(ax), \cos(bx)$ use:

(d) $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

Evaluate $\int \sin(5x)\cos(3x)dx$.

To integrate $\int \tan^k x \sec^j x dx$:

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.
(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$. Then use $u = \sec x$.
(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.
(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \tan^6 x \sec^4 x dx$.

To integrate $\int \tan^k x \sec^j x dx$:

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.
(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 -1$. Then use $u = \sec x$.
(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.
(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \tan^3 x \sec^3 x dx$.

Verify $\int \sec x dx = \ln(\sec x + \tan x) + C$

Evaluate $\int \sec^3 x dx$.

Section 3.3: Integration involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$.

Some such integrals we can already do:

$$\int \frac{dx}{\sqrt{a^2-x^2}}, \qquad \int \frac{x\,dx}{\sqrt{a^2-x^2}}, \qquad \int x\sqrt{a^2-x^2}\,dx$$

Integrals involving $\sqrt{a^2 - x^2}$.

Use substitution $x = a\sin\theta$, $dx = a\cos\theta d\theta$.
Converts $\sqrt{a^2 - x^2}$ to $a\cos\theta$.
Simplify.
Evaluate.
Use reference triangle to convert $\theta = \sin^{-1}(x/a)$ back to x.

Integrals involving $\sqrt{a^2 - x^2}$, use substitution $x = a\sin\theta$.

Evaluate $\sqrt{9 - x^2}$.

Integrals involving $\sqrt{a^2 - x^2}$, use substitution $x = a\sin\theta$.

Evaluate $\int \frac{1}{x}\sqrt{4 - x^2}dx$.

Integrands involving $\sqrt{a^2+x^2}$.

Check for alternate methods.
Substitute $x = a\tan\theta$ and $dx = a\sec^2\theta d\theta$.
Converts $\sqrt{a^2+x^2}$ to $a\sec\theta$.
Simplify.
Evaluate trig integral.
Use reference triangle to convert back to x.

Integrals involving $\sqrt{a^2 + x^2}$, use substitution $x = a\tan\theta$.

Evaluate $\int \frac{dx}{\sqrt{1+x^2}}$

Integrals involving $\sqrt{a^2 + x^2}$, use substitution $x = a \tan \theta$.

Find the arclength of $y = x^2$ over $[0, 1]$.

Integrands involving $\sqrt{x^2 - a^2}$.

Check for alternate methods.
Use $x = a\sec\theta$ and $dx = a\sec\theta\tan\theta d\theta$.
Simplify.
Evaluate.
Use reference triangle to convert to x.

Integrals involving $\sqrt{x^2 - a^2}$, use substitution $x = a \sec \theta$.

Evaluate $\int_3^5 \sqrt{x^2 - 9} dx$

Quiz 8 review: Only Sections 3.1 and 3.2.

Page 1: Integration by parts.

- 2 simple integration by parts
- 2-part problem: use $dv = 1$, integrate by parts twice and solve for integral
- integrate by parts using $dv = 1$ and $\ln x$.

Page 2: Trigonometric integrals.

- 3 problems: choose correct strategy from sheet.
- 2 questions: evaluate 2 trig power integrals

Evaluate $\int x \cos x dx$

(1) Apply integration by parts to $\int \sin x e^x dx$.

(2) Apply integration by parts again and solve for the integral above.

Evaluate $\int \ln \sqrt{x} dx$.

What is the correct strategy to evaluate: $\int \sin(9x)\cos(8x)dx$?

(a) Replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) Use $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.

Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.

Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

What is the correct strategy to evaluate: $\int \cos^7 x \sin^2 x dx$?

(a) Replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

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(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.

Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.

Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

What is the correct strategy to evaluate: $\int \tan^5 x \sec^2 x dx$?

(a) Replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) Use $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.

Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.

Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \sin^5 x \cos^5 x dx$

Evaluate $\int \tan^2 x \sec^6 x dx$