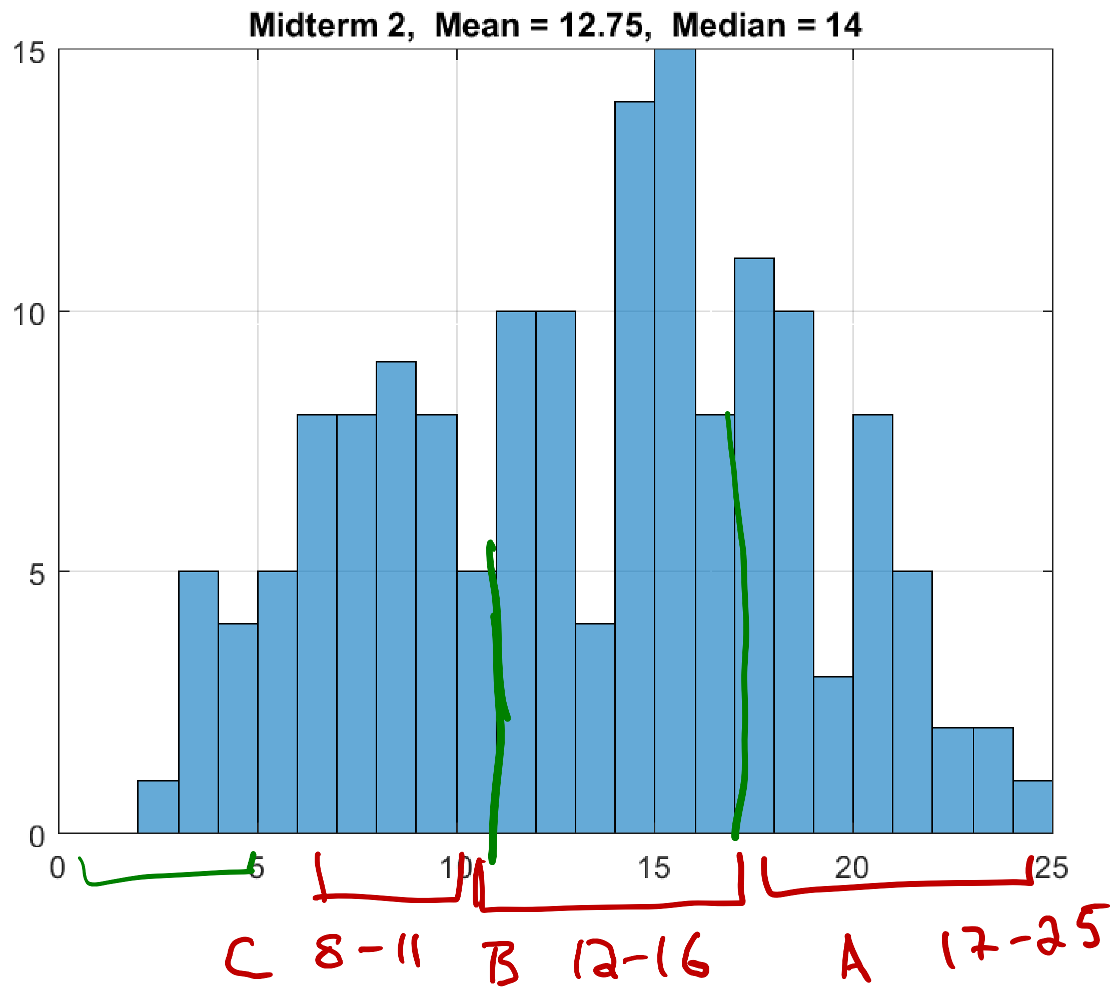
MAT 126.01, Prof. Bishop, Tuesday, Oct 27, 2020
Section 3.1: Integration by parts
Section 3.2: Trigonometric Integrals ←

① HW 9 was adjusted

② Next Tuesday recorded class.

③ Next week's quiz, allowed 1 page of notes

Quiz 8

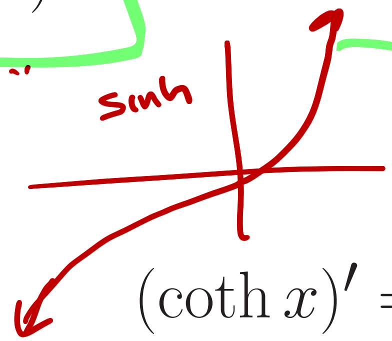


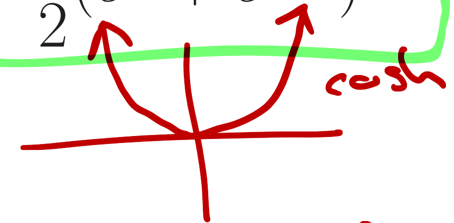
Midterm 2, Mean = 12.75, Median = 14
15
10
5
0
0
5
10
15
20
25
C 8-11
B 12-16
A 17-25

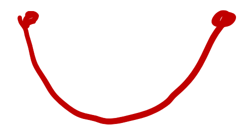
Section 2.9: Hyperbolic trig functions (not on HW or exams).

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

"hyperbolic sin"







$$(\sinh x)' = \cosh x$$
$$(\cosh x)' = \sinh x$$
$$(\tanh x)' = \text{sech}^2 x$$

$$(\coth x)' = -\text{csch} x$$
$$(\text{sech} x)' = -\text{sech} x \tanh x$$
$$(\text{csch} x)' = -\text{csch} x \coth x$$

$$(\sinh^{-1} x)' = 1/\sqrt{1+x^2}$$
$$(\cosh^{-1} x)' = 1/\sqrt{x^2-1}$$
$$(\tanh^{-1} x)' = 1/(1-x^2)$$

$$(\coth^{-1} x)' = 1/(1-x^2)$$
$$(\text{sech}^{-1} x)' = -1/x\sqrt{1-x^2}$$
$$(\text{csch}^{-1} x)' = -1/|x|\sqrt{1+x^2}$$

$$\int \frac{1}{\sqrt{1+x^2}} = \sinh^{-1}(x) + C$$

Product rule:

$$(fg)' = f'g + fg'$$

$$\int (fg)' = \int f'g + \int fg'$$

$$fg = \int f'g + \int fg'$$

$$fg - \int f'g = \int fg'$$

Integration by parts:

$$\int u dv = uv - \int v du$$

Evaluate: $\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv}$.

what is u? → du

what is dv? → v

$u = e^x \quad dv = x\,dx$

$du = e^x dx \quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 e^x - \int \frac{1}{2}x^2 e^x dx$

not helpful

$\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv} = uv - \int v\,du$

$u = x$

$du = 1 \cdot dx$

$dv = e^x$

$v = e^x$

$= xe^x - \int e^x \cdot 1 \cdot dx$

$= xe^x - e^x + C$ ✓

diff: $1 \cdot e^x + xe^x - e^x = xe^x$

Evaluate: $\int x \cos x dx$.

$u = x \qquad dv = \cos x \, dx$

$du = 1 \cdot dx \qquad v = \sin x$

$$\int x \cos x \, dx = uv - \int v \, du$$

$$= x \sin x - \int \sin x \cdot 1 \cdot dx$$

$$= x \sin x - (-\cos x)$$

$$= x \sin x + \cos x + C$$

$$(\quad)' = 1 \cdot \sin x + x \cos x - \sin x$$

Evaluate: $\int x^{-3} \underbrace{\ln x}_{u} dx$.

$u = \ln x \qquad dv = x^{-3} dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{-2} x^{-2}$

$= uv - \int v\, du$

$= (\ln x) \cdot \left(-\frac{1}{2} x^{-2}\right) - \int \left(-\frac{1}{2}\right) x^{-2} \frac{1}{x} dx$

$= -\frac{\ln x}{2x^2} + \frac{1}{2} \int x^{-3} dx$

$= \frac{-\ln x}{2x^2} + \frac{1}{2}\left(\frac{1}{-2} x^{-2}\right) + C$

$= \frac{-\ln x}{2x^2} - \frac{1}{4} \frac{1}{x^2} + C$

Sometimes we need to use it more than once.

Evaluate $\int x^2 \sin x dx$.

$u = x^2 \qquad du = 2x\,dx$

$dv = \sin x\, dx \qquad v = -\cos x$

$= uv - \int v\, du = x^2(-\cos x) - \int (-\cos x)\, 2x\, dx$

$= -x^2 \cos x + 2 \int x \cos x\, dx$

$\int x \cos x\, dx = x \cdot \sin x - \int \sin x \, dx$

$= x \sin x + \cos x$

$= -x^2 \cos x + 2 [x \sin x + \cos x] + C$

LIATE: first choices for u:

Logarithmic Functions,
Inverse Trigonometric Functions,
Algebraic Functions,
Trigonometric Functions,
Exponential Functions.

Sometimes it is a good idea to take $dv = 1$.

Evaluate $\int \ln x dx$

$$\int 1 \cdot \underbrace{\ln x}_{u} \, dx = x \ln x - \underbrace{\int x \frac{1}{x} dx}_{\underbrace{\int 1 dx}_{x}}$$

$u = \ln x \qquad dv = 1 \cdot dx$

$du = \frac{1}{x} dx \qquad v = x$

$$= x \ln x - x + C$$

A trickier one:

Evaluate $\int t^3 e^{t^2} dt \quad = \int (t^2)(t e^{t^2})\,dt$

$u = t^2$, $dv = t e^{t^2} dt$

$\int t e^{t^2} dt$

$u = t^2$

$du = 2t\,dt$

$dv = t e^{t^2} dt$

$v = \frac{1}{2} e^{t^2}$

$\int t e^{t^2} dt$

$u = t^2$

$du = 2t\,dt$

$= \frac{1}{2} \int e^u du$

$= \frac{1}{2} e^u$

$= \frac{1}{2} e^{t^2}$

$= uv - \int v\,du$

$= t^2 \frac{1}{2} e^{t^2} - \int \frac{1}{2} e^{t^2} 2t\,dt$

$= \frac{t^2}{2} e^{t^2} - \int t e^{t^2} dt$

$= \frac{t^2}{2} e^{t^2} - \frac{1}{2} e^{t^2} + C$

Sometimes we integrate twice to get the same expression.

Evaluate $\int \sin x e^x dx$.

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

$= uv - \int v \, du$

$= \sin x e^x - \int e^x \cos x \, dx$

$= \sin x e^x - \left[\cos x \cdot e^x - \int e^x (-\sin x) dx \right]$

$= (\sin x) e^x - (\cos x) e^x - \int e^x \sin x \, dx$

$$\frac{2 \int \sin x e^x dx}{2} = \frac{\sin x e^x}{2} - \frac{\cos x e^x}{2}$$

Some time we use both $dv = 1$ and integrate twice:

Evaluate $\int \sin \ln x dx$

$$\int \underbrace{\sin \ln x}_{u} \cdot \underbrace{1\, dx}_{dv} = x \sin(\ln x) - \int x \frac{\cos \ln x}{x} dx$$

$$du = \frac{\cos \ln x}{x} \qquad v = x$$

$$= x \sin(\ln x) - \Big[x \cos \ln x - \int$$

$$u = \cos \ln x \qquad dv = 1\, dx$$

$$du = -\sin l$$

Section 3.2 Trigonometric integrals

Integrating powers of trig functions.

Several different strategies to use depending on the form of the integrand.

There is a list on the class webpage that you may print and bring to next week's quiz.

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \cos^3 x \sin x dx$

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \sin^4 x dx$

Evaluate $\int \cos^4 \sin^3 x dx$

To integrate products of $\sin(ax), \sin(bx), \cos(ax), \cos(bx)$ use:

(d) $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

Evaluate $\int \sin(5x)\cos(3x)dx$.

To integrate $\int \tan^k x \sec^j x dx$:

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 -1$. Then use $u = \sec x$.

(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.

(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \tan^6 x \sec^4 x dx$.

Evaluate $\int \tan^3 x \sec^3 x dx$.

Verify $\int \sec x dx = \ln(\sec x + \tan x) + C$

Evaluate $\int \sec^3 x dx$.