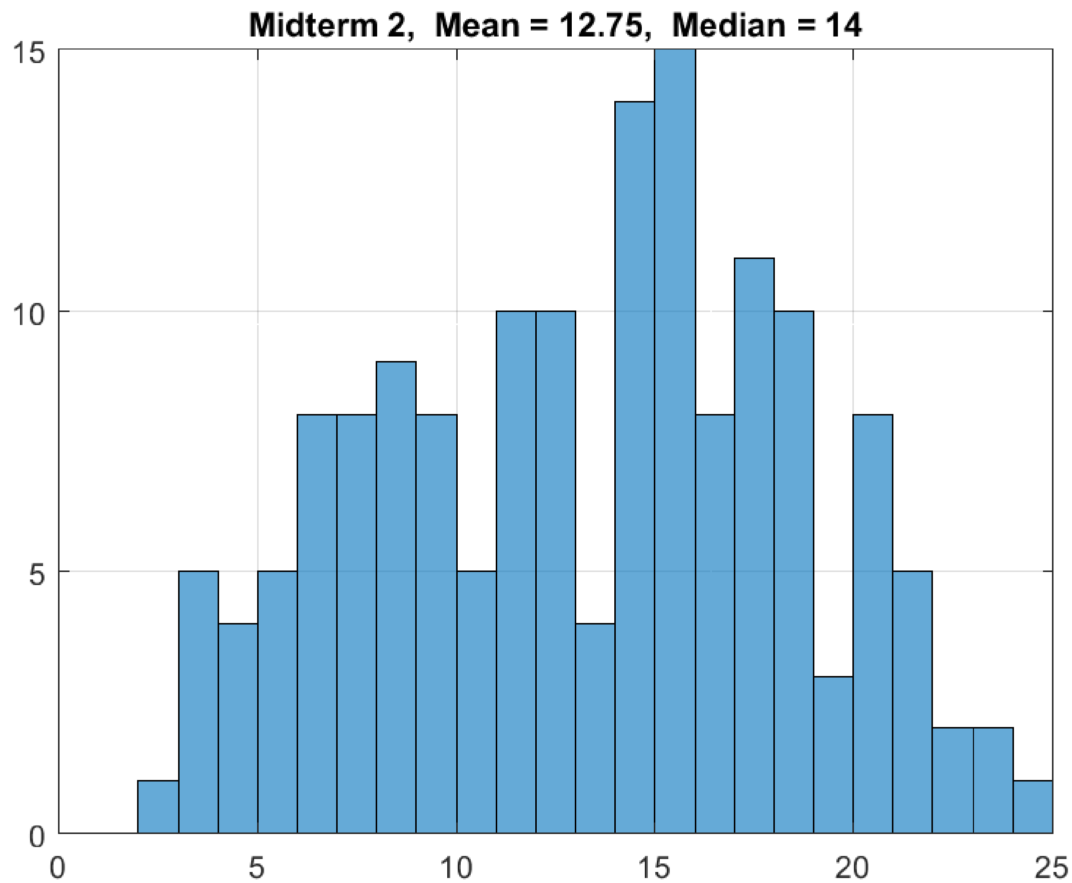
MAT 126.01, Prof. Bishop, Tuesday, Oct 27, 2020
Section 3.1: Integration by parts
Section 3.2: Trigonometric Integrals


Midterm 2, Mean = 12.75, Median = 14
15
10
5
0
0
5
10
15
20
25

Section 2.9: Hyperbolic trig functions (not on HW or exams).

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$(\sinh x)' = \cosh x$$
$$(\cosh x)' = \sinh x$$
$$(\tanh x)' = \text{sech}^2 x$$

$$(\coth x)' = -\text{csch} x$$
$$(\text{sech} x)' = -\text{sech} x \tanh x$$
$$(\text{csch} x)' = -\text{csch} x \coth x$$

$$(\sinh^{-1} x)' = 1/\sqrt{1+x^2}$$
$$(\cosh^{-1} x)' = 1/\sqrt{x^2-1}$$
$$(\tanh^{-1} x)' = 1/(1-x^2)$$

$$(\coth^{-1} x)' = 1/(1-x^2)$$
$$(\text{sech}^{-1} x)' = -1/x\sqrt{1-x^2}$$
$$(\text{csch}^{-1} x)' = -1/|x|\sqrt{1+x^2}$$

Product rule:

$$(fg)' = f'g + fg'$$

$$\int (fg)' = \int f'g + \int fg'$$

$$fg = \int f'g + \int fg'$$

$$fg - \int f'g = \int fg'$$

Integration by parts:

$$\int udv = uv - \int vdu$$

Evaluate: $\int xe^x dx$.

Evaluate: $\int x \cos x dx$.

Evaluate: $\int x^{-3} \ln x dx$.

Sometimes we need to use it more than once.

Evaluate $\int x^2 \sin x dx$.

LIATE: first choices for u:

Logarithmic Functions,
Inverse Trigonometric Functions,
Algebraic Functions,
Trigonometric Functions,
Exponential Functions.

Sometimes it is a good idea to take $dv = 1$.

Evaluate $\int \ln x dx$

A trickier one:

Evaluate $\int t^3 e^{t^2} dt$

Sometimes we integrate twice to get the same expression.

Evaluate $\int \sin x e^x dx$.

Some time we use both $dv = 1$ and integrate twice:

Evaluate $\int \sin \ln x dx$

Section 3.2 Trigonometric integrals

Integrating powers of trig functions.

Several different strategies to use depending on the form of the integrand.

There is a list on the class webpage that you may print and bring to next week's quiz.

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \cos^3 x \sin x dx$

To integrate $\int \cos^j x \sin^k x dx$:

(a) If k is odd replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) If j is odd, replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) If both j and k are even, use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

Evaluate $\int \sin^4 x dx$

Evaluate $\int \cos^4 \sin^3 x dx$

To integrate products of $\sin(ax), \sin(bx), \cos(ax), \cos(bx)$ use:

(d) $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

Evaluate $\int \sin(5x)\cos(3x)dx$.

To integrate $\int \tan^k x \sec^j x dx$:

(g) If j is even, and $j \geq 2$ rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) If k is odd and $j \geq 1$, rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$. Then use $u = \sec x$.

(i) If k is odd, $k \geq 3$ and $j = 0$, rewrite $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$. Repeat if necessary.

(j) If k is even and j is odd, then use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

Evaluate $\int \tan^6 x \sec^4 x dx$.

Evaluate $\int \tan^3 x \sec^3 x dx$.

Verify $\int \sec x dx = \ln(\sec x + \tan x) + C$

Evaluate $\int \sec^3 x dx$.