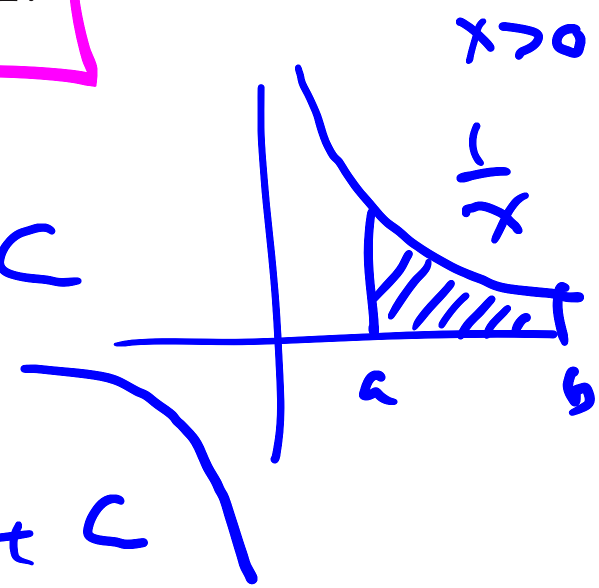
MAT 126.01, Prof. Bishop, Tuesday, Oct 20, 2020
Last minute review for midterm
Section 2.7: Integrals, Exponentials and Logarithms

Any last minute questions about Midterm 2?

The power law for integrals:

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \boxed{n \neq -1.}$$

$$\int x^5 = \frac{1}{6}x^6 + C$$



$$\int x^{-3} = \frac{1}{-2}x^{-2} + C$$

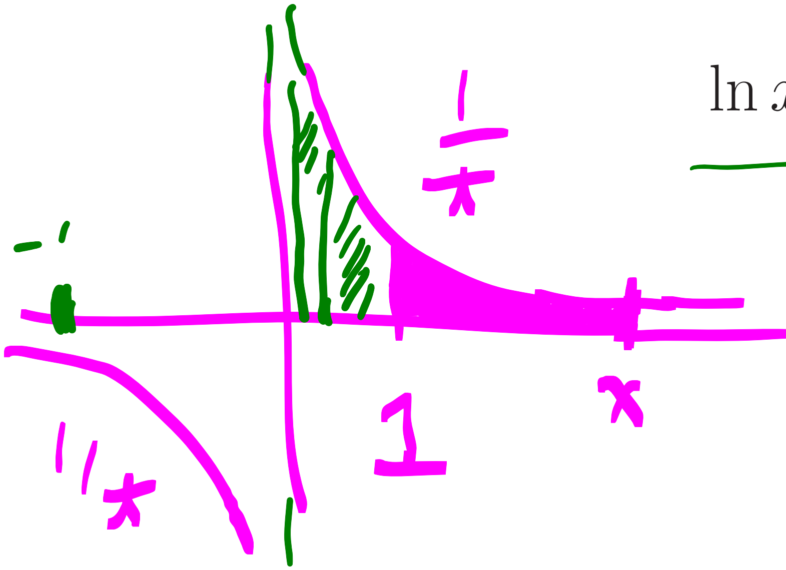
$$\int \frac{1}{x} = \int x^{-1} = ? = \frac{1}{-1+1}x^{-1+1}$$

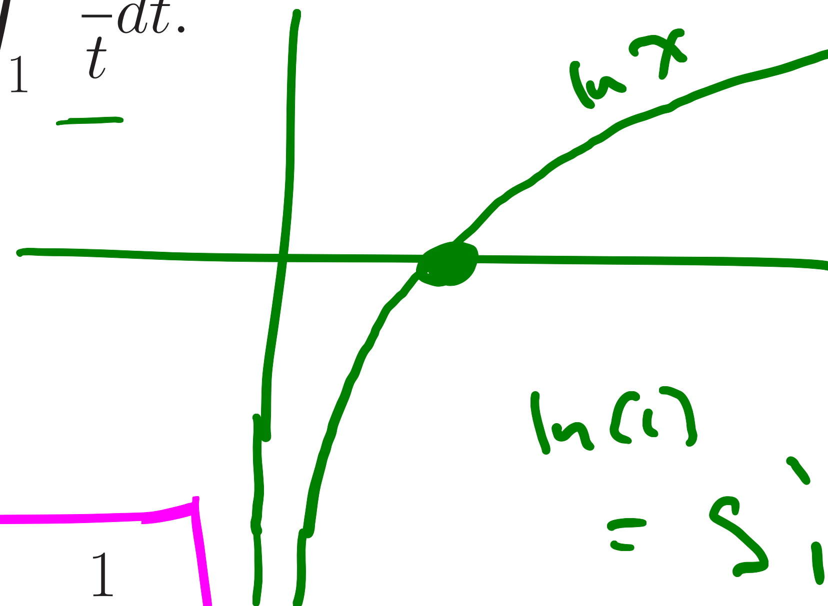
$$= \frac{1}{0}x^0 ?$$

Definition of natural logarithm:

$x > 0$

$$\ln x = \int_1^x \frac{1}{t}\,dt.$$





$\ln(1) = \int_1^1 \frac{1}{t}\,dt$

Differentiation law:

$$\boxed{(\ln x)' = \frac{1}{x}.}$$

Properties:

$$\ln 1 = 0$$

$\ln 1 = \int_1^1 \frac{1}{t}\, dt = 0$

$$\ln(ab) = \ln a + \ln b$$

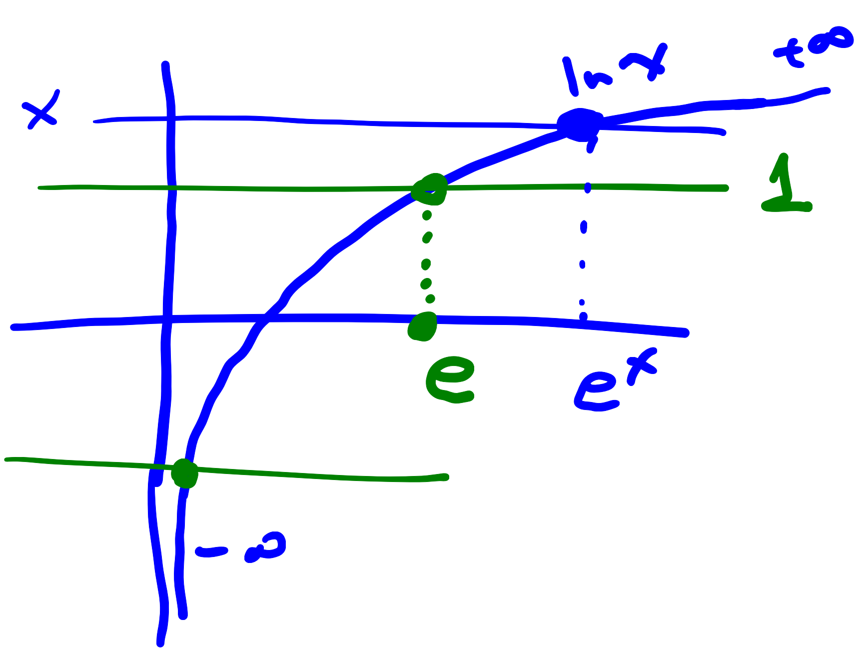
$$\ln(a/b) = \ln a - \ln b$$

$$\ln a^p = p \ln a.$$

Definition of e: e is the unique number so $\ln e = 1$.

$e \approx \underline{2.71\ldots}$

e^x is defined as the unique number y so $\ln y = x$.



$$\ln x = \int_1^x \frac{1}{t}\,dt$$

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

Properties of exponential function:

$$e^{x+y} = e^x \cdot e^y$$

$$e^{x-y} = e^x / e^y$$

$$(e^x)^p = e^{xp}.$$

$$\ln(e^{x+y}) \quad\quad \ln(e^x \cdot e^y)$$

$$\downarrow \quad\quad \downarrow$$

$$x+y \quad = \quad \ln(e^x) + \ln(e^y)$$

$$! \quad\quad x + y$$

General exponential functions:

If $a > 0$ define $a^x = e^{x \ln a}$

$$\boxed{2^x = e^{x \ln 2}}$$

$$= \left(e^{\ln 2}\right)^x = 2^x$$

Deduce $(a^x)' = a^x \ln a$

$$2^\pi = ? = e^{\pi \ln 2}$$

$$2^3 = 2 \cdot 2 \cdot 2$$

$$(a^x)' = \left(e^{x \ln a}\right)'$$

$$= e^{x \ln a} \cdot (x \ln a)'$$

$$= e^{x \ln a} \cdot \ln a = a^x \cdot \underbrace{\ln a}_{a=e,\ =1}$$

Deduce $\int a^x = (a^x / \ln a) + C$

General logarithmic functions:

Define $\log_a x = \ln x / \ln a$.

$\log_e = \ln$, $\log_{10} x = \frac{\ln x}{\ln 10}$, $10^{\log_{10} x} = x$

Deduce $(\log_a x)' = 1/(x \ln a)$.

$$(\log_a x)' = \left(\frac{\ln x}{\ln a}\right)' = \frac{1}{x} \cdot \frac{1}{\ln a}$$

Differentiate $\ln \ln \ln x$.

1 2 3

$$(\underbrace{\ln}\,(\underbrace{\ln}\,(\underbrace{\ln}\, x)))' = \frac{1}{\ln \ln x} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$f(g(x))' = f'(g(x)) \cdot g'(x)$$

$$\rightarrow \underbrace{f(g(\underbrace{h(x)}))}' = \underbrace{f'(g(h(x)))}_{\text{outside}} \cdot \underbrace{g'(h(x)) \cdot h'(x)}_{\text{inside}}$$

Differentate $\ln(x + \sqrt{1 + x^2})$

$$= \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \cancel{\tfrac{1}{2}}(1+x^2)^{-\frac{1}{2}} \cdot \cancel{2}x\right)$$

$$= \frac{1}{x+\sqrt{1+x^2}} \left(1 + \frac{x}{\sqrt{1+x^2}}\right)$$

$$= \frac{1}{\boxed{x+\sqrt{1+x^2}}} \cdot \frac{\boxed{\sqrt{1+x^2} + x}}{\sqrt{1+x^2}}$$

$$= \frac{1}{\sqrt{1+x^2}}$$

later on

$$\boxed{\int \frac{1}{\sqrt{1+x^2}} = \ln\left[x+\sqrt{1+x^2}\right]}$$

Differentiate $x^{\sin x}$.

WRONG $\longrightarrow$ $\cancel{x^{\sin x - 1} \cdot \sin x}$

Right $\longrightarrow$ $\left(\underbrace{e^{\ln x}}_{=x}\right)^{\sin x} = e^{(\ln x)(\sin x)}$

$$[x^{\sin x}]' = [e^{\ln x \sin x}]'$$

$$= e^{\ln x \sin x} \cdot \left[\frac{1}{x} \sin x + \ln x \cos x\right]$$

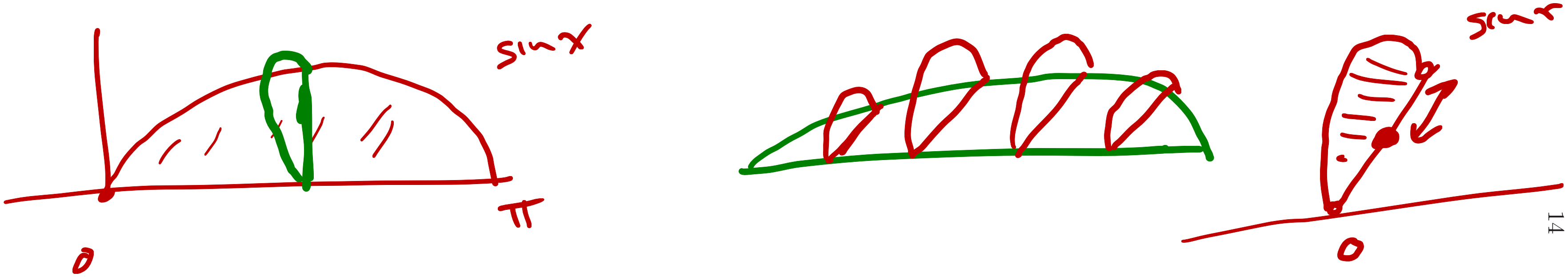
$$= x^{\sin x} \left[\frac{1}{x} \sin x + \ln x \cos x\right]$$

Differentiate $(\sin x)^x$ for $0 < x < \pi$.

$$(\sin x)^x = e^{\ln(\sin x)\cdot x}$$

$$[(\sin x)^x]' = e^{\ln(\sin x)\cdot x} \cdot \left[\frac{1}{\sin x}\cdot \cos x \cdot x + \ln \sin x \cdot 1\right]$$

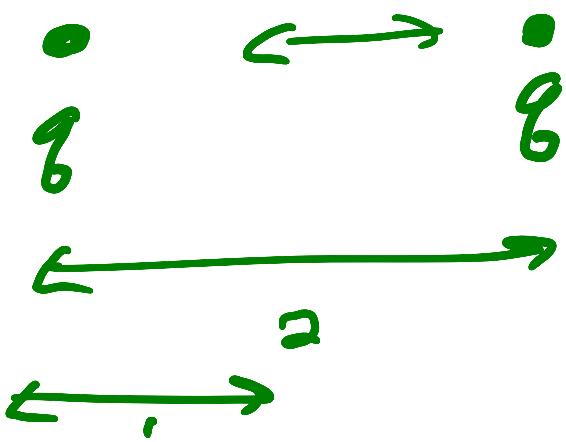
$$= (\sin x)^x \cdot \left[\cot x \cdot x + \ln \sin x\right]$$



$$\text{Vol} = \int_0^{\pi} \frac{\pi}{2}\Big(\underbrace{\frac{\sin x}{2}}_{r}\Big)^2 dx$$

Area half circle $= \dfrac{\pi r^2}{2}$

Quiz 6, Prob 10



$$\boxed{\int_2^1 \frac{kqq}{x^2}\,dx}$$

$$= \boxed{kq^2} \int_2^1 \frac{1}{x^2}\,dx$$

$$= kq^2 \left. -\frac{1}{x} \right|_2^1$$

$$= kq^2 \left(-1 - \left(-\frac{1}{2}\right)\right)$$

$$= -kq^2 \frac{1}{2}$$

$$\boxed{kq^2/2}$$

Quiz 4 Prob 7

$$\int \frac{1}{x + x\ln^2 x}\,dx$$

$$= \int \frac{1}{x} \cdot \frac{1}{1+\ln^2 x}\,dx$$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$$= \int \frac{1}{1+u^2}\,du$$

$$= \tan^{-1}(u)$$

$$= \tan^{-1}(\ln x) + C$$

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MAT 126 Tue Oct 20

Office Hours start ≈ 11:20

HW 8, Prob 12

$y = 2x - 4x^2$

$2x - 4x^2 = 0$

$2x(1-2x) = 0$

$x = 0 \quad x = 1/2$

① Find region ✓

② Find total area: ✓

$$\int_0^{1/2} 2x - 4x^2 \, dx = x^2 - \frac{4}{3}x^3 \Big|_0^{1/2}$$

$$= \frac{1}{4} - \frac{4}{3}\cdot\frac{1}{8} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

③ Find intersection point p:

$$2x-4x^2 = \star x \qquad \text{solve for } x$$

$$2x-\star x = 4x^2$$

$$2-\star = 4x$$

$$\frac{2-\star}{4} = x$$

④ Find area between parabola & line

$$\int_0^{2-\star/4} (2x-4x^2)-(\star x)\,dx$$

$$(2-\star)x-4x^2$$

$$\frac{2-\star}{2}x^2-\frac{4}{3}x^3\Big|_0^{\frac{2-\star}{4}}$$

$$=\underbrace{2\cdot\frac{2-\star}{4}\cdot\left(\frac{2-\star}{4}\right)^2}-\frac{4}{3}\left(\frac{2-\star}{4}\right)^3$$

$$=\left(\frac{2-\star}{4}\right)^3\cdot\left(\underset{\frac{2}{3}}{2-\frac{4}{3}}\right)$$

(5)

$$\cancel{\frac{2}{3}}\left(\frac{2-x}{4}\right)^3 = \frac{1}{2}\cdot\frac{1}{\cancel{4\cdot 3}\;8}$$

$$\frac{2-x}{4} = \left(\frac{1}{2}\cdot\frac{1}{8}\right)^{1/3} = \frac{1}{2}\cdot 2^{-1/3}$$

$$2-x = 2\;2^{-1/3}$$

$$x = 2 - 2\cdot 2^{-1/3}$$

$$= 2\left(1-2^{-1/3}\right)$$