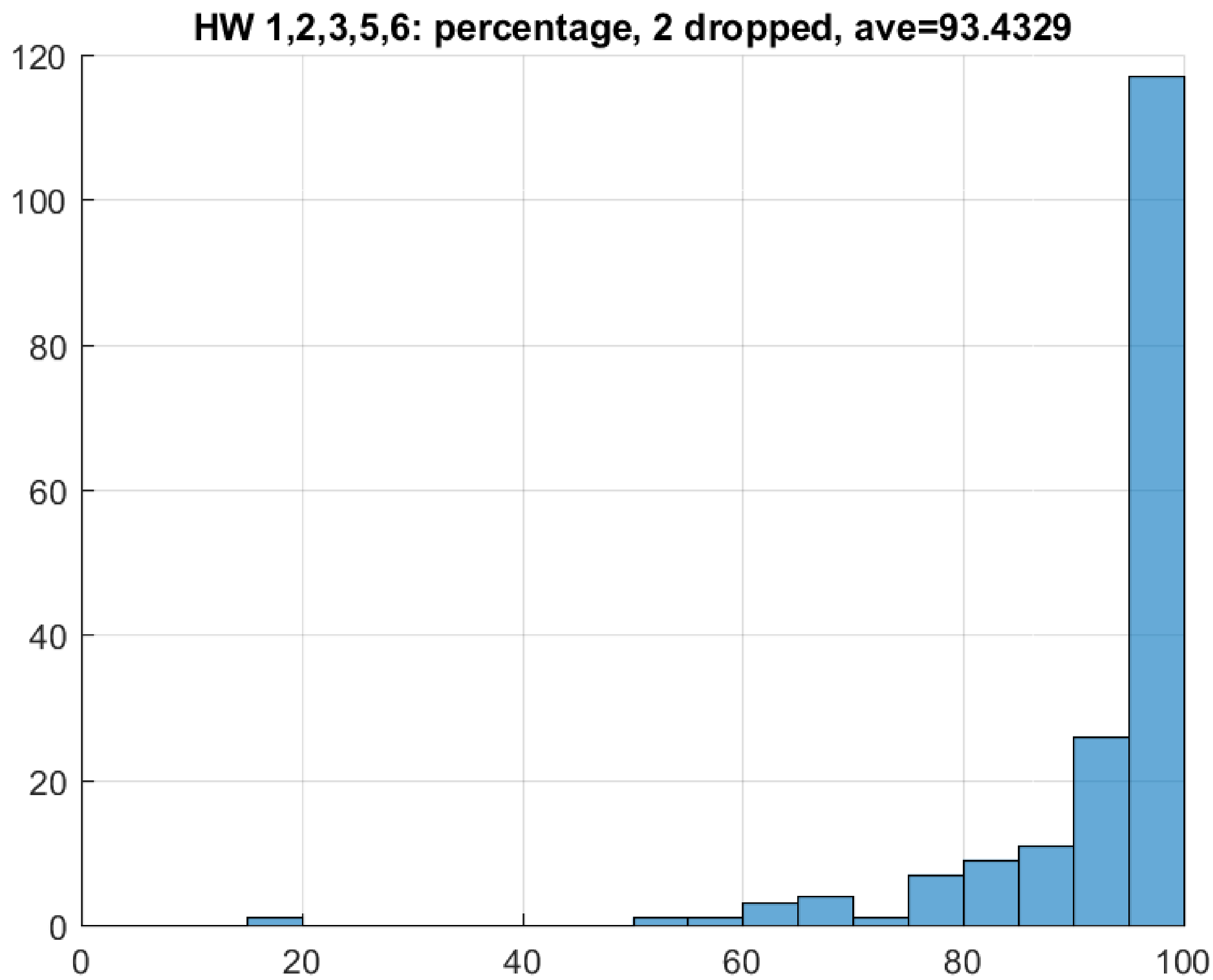
MAT 126.01, Prof. Bishop, Thursday, Oct 15, 2020
Midterm 2 review


HW 1,2,3,5,6: percentage, 2 dropped, ave=93.4329
120
100
80
60
40
20
0
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20
40
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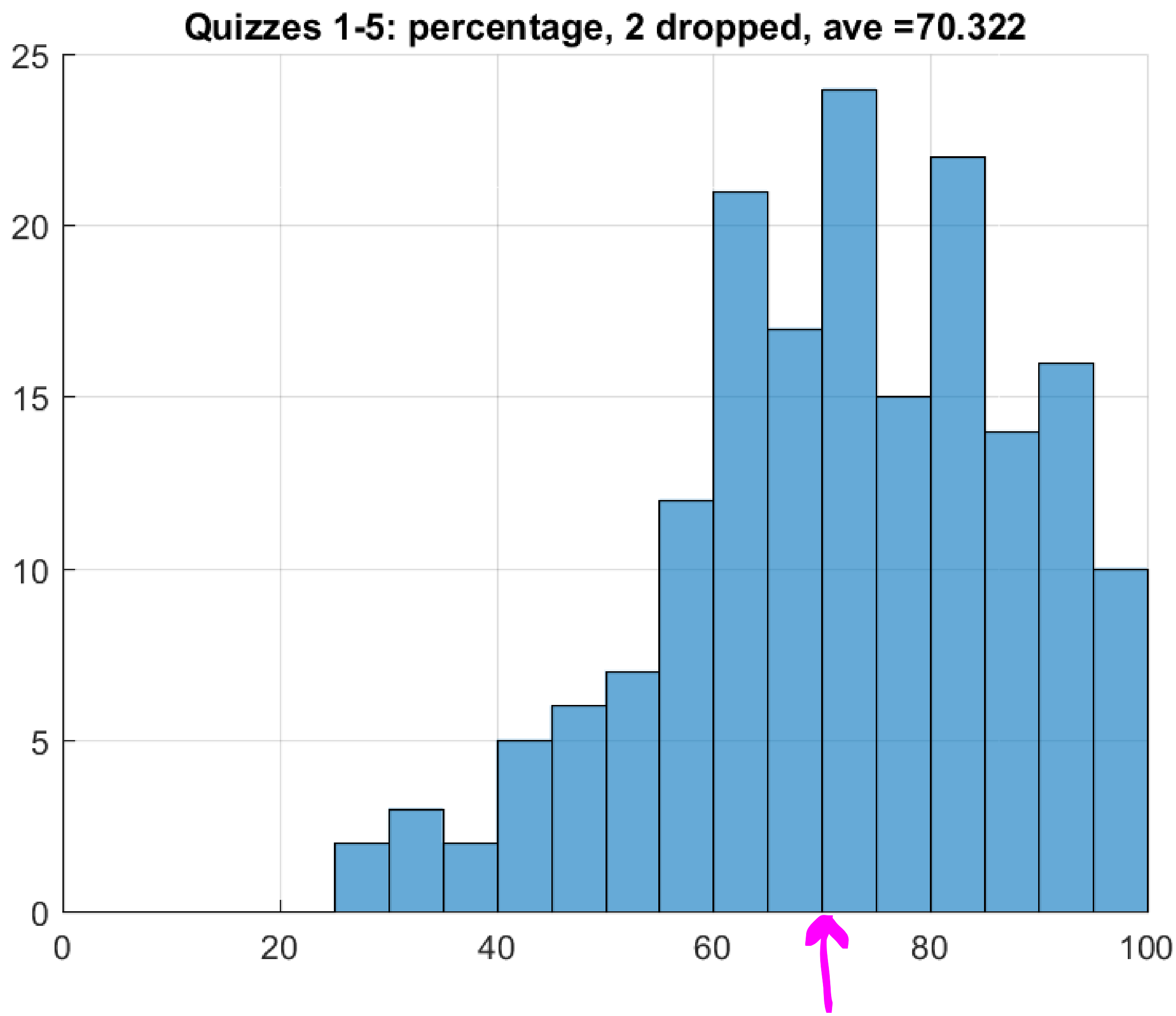
80
100



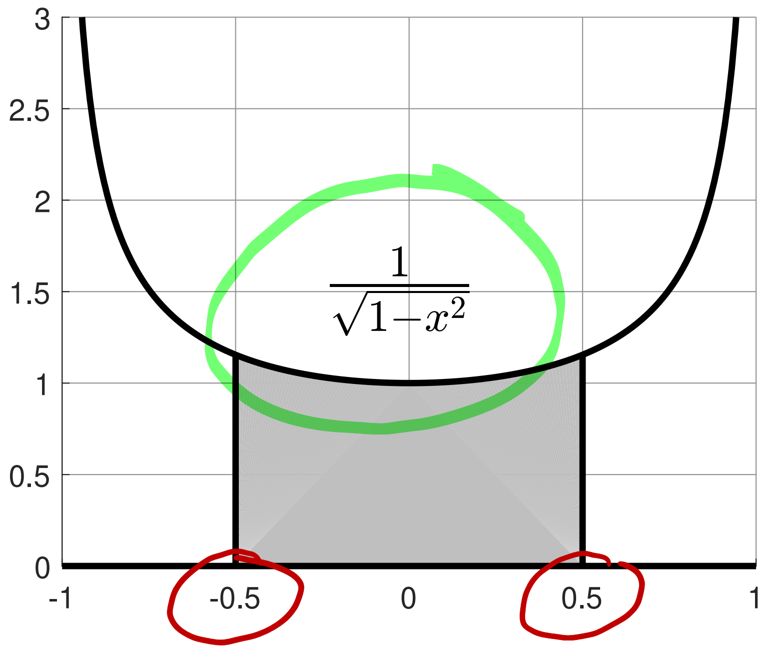
Midterm 2: 25 multiple choice questions on 6 pages.

- Page 1: 4 integrations involving exponential, logarithms, inverse trig functions. Q 4
- Page 2: 2 matching formulas/figures (area between graphs), 2 computing areas. Q 4
- Page 3: 4 matching formulas/figures, volumes of revolution (Q 5
- Page 4: 4 problems on volumes of revolution (2 disks, 2 shells) Q 5
- Page 5: 5 problems on volume, arclength and area. Q 6
- Page 6: is 4 problems on physical applications (lifting and population).

Q 6

xe^{-x}

Compute the area of the shaded region.



① $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

$a = 1$

$= \sin^{-1}(x) \Big|_{-1/2}^{1/2} = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right)$

$= 2\,\frac{\pi}{6}$

$\boxed{= \pi/3}$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$

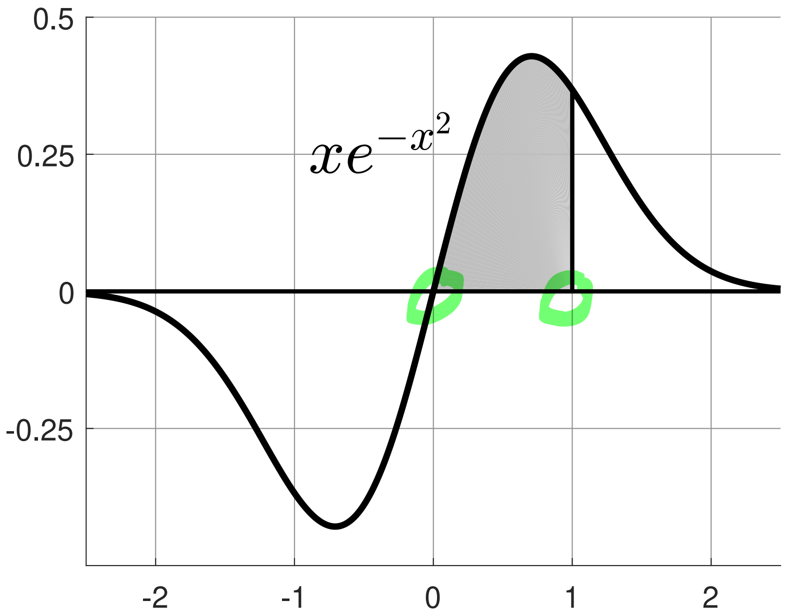
$\sin^{-1}\left(-\frac{1}{2}\right) = -\pi/6$

Section 1.7
page 106

② $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

③ $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$

Compute the area of the shaded region.



$$\int_0^1 x e^{-x^2} dx$$

$$u = -x^2$$

$$du = -2x\,dx$$

$$\frac{1}{-2} \int (-2x) e^{-x^2} dx$$

$$-\frac{1}{2} \int e^u du$$

$$-\frac{1}{2} e^u$$

$$-\frac{1}{2} e^{-x^2} \Big|_0^1$$

$$= -\frac{1}{2} \left[e^{-1} - e^0 \right]$$

$$\boxed{= \frac{1}{2} \left[1 - e^{-1} \right]}$$

Compute the indefinite integral of $f(x) = \tan(x) \ln \sec(x)$.

$\sec = \frac{1}{\cos}$

$(\sec)' = \frac{1}{\cos}$

$= \frac{0 - (1)(-\sin)}{\cos^2}$

$= \frac{\sin}{\cos^2}$

$= \frac{\sin}{\cos} \cdot \frac{1}{\cos}$

$= \tan \cdot \sec$

$\cancel{u = \sec x}$

$\cancel{du = \sec x} \; \tan x \, dx$

$u = \ln \sec(x)$

$du = \frac{1}{\cancel{\sec}} \cdot \cancel{\sec} \cdot \tan$

$= \tan$

$\int \underbrace{\tan(x)}_{du} \underbrace{\ln \sec x}_{u}$

$= \int u \, du$

$= \frac{1}{2} u^2$

$\frac{1}{2} \ln^2(\sec(x)) \longleftrightarrow = \frac{1}{2}(\ln \sec(x))^2$

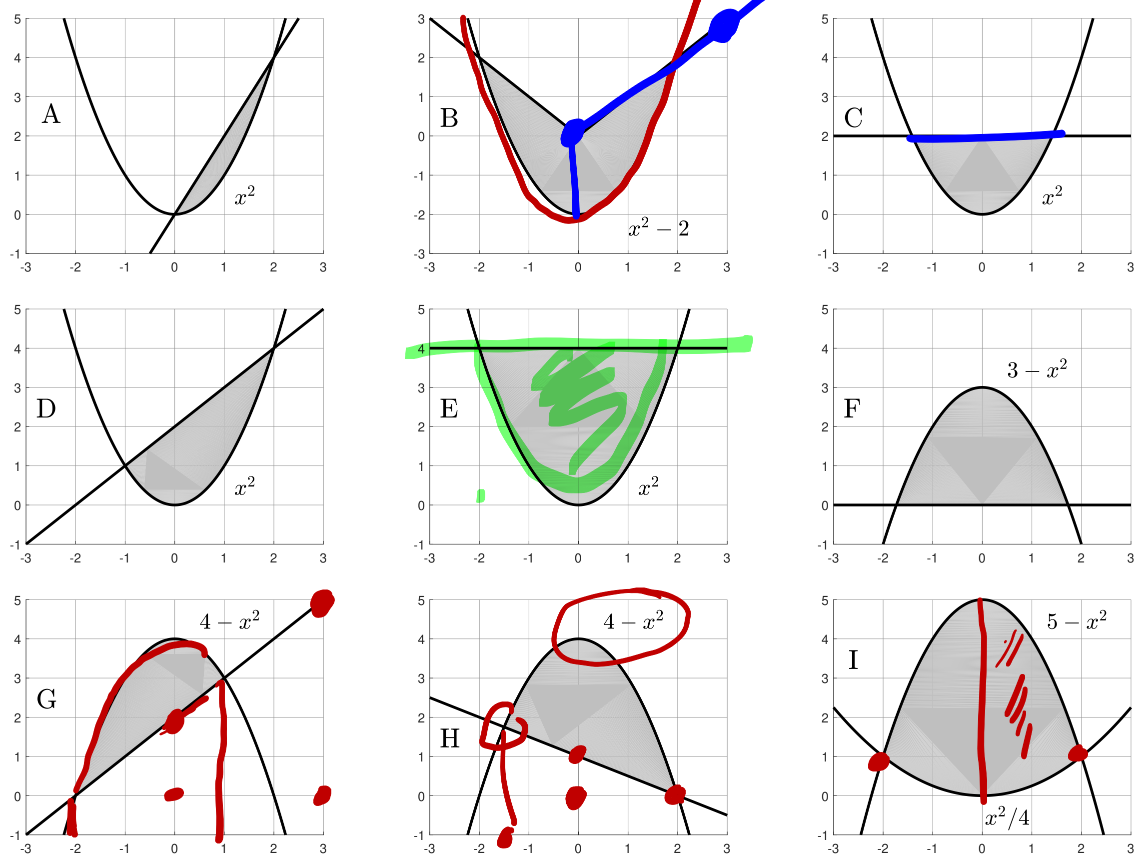
Page 2

Match each formula for the area to the region it describes.

B $2\int_0^2 2+x-x^2dx$ E $\int_{-2}^2 4-x^2dx$

$x-(x^2-2) = x+2-x^2$

x



$2-x^2$

$2\cdot\int_0^2$

G: $\int_{-2}^1 4-x^2-(x+2)\,dx$

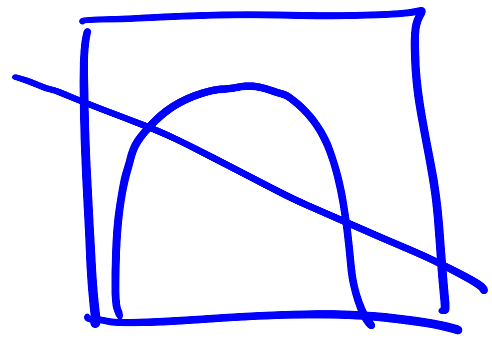
$= \int_{-2}^1 2-x-x^2 = 2x-\frac{1}{2}x^2-\frac{1}{3}x^3\Big|_{-2}^{1}$

I $\int_{-2}^2 (5-x^2)-(\frac{x^2}{4})$

$= \int_{-2}^2 5-\frac{5}{4}x^2$

$= \left[5x-\frac{5}{12}x^3\right]_{-2}^2$

Compute the shaded area of picture H.



$$y = 4 - x^2$$
$$y = 1 - \tfrac{1}{2}x$$

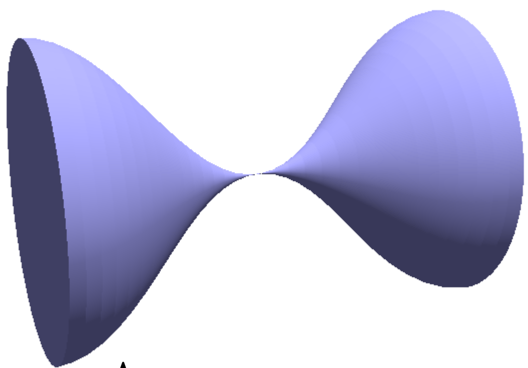
$$4 - x^2 = 1 - \tfrac{1}{2}x$$
$$0 = x^2 - \tfrac{1}{2}x - 3$$
$$= \tfrac{1}{2}(2x^2 - x - 6)$$
$$= \tfrac{1}{2}(2x+3)(x-2)$$
$$x = -3/2 \qquad x = 2$$

$$\int_{-3/2}^{2} (4 - x^2) - (1 - \tfrac{1}{2}x)$$
$$= \int_{-3/2}^{2} \left(3 + \tfrac{1}{2}x - x^2\right)$$
$$= \left[3x + \tfrac{1}{4}x^2 - \tfrac{1}{3}x^3\right]_{-3/2}^{2}$$
$$= \left[6 + 1 - \tfrac{8}{3}\right] - \left[-\tfrac{9}{2} + \tfrac{9}{16} + \tfrac{27}{24}\right]$$
$$=$$

Page 3 Q5

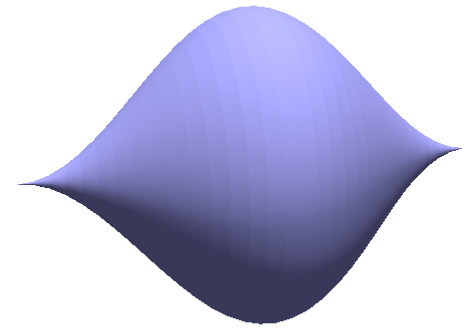
Match each formula with the picture of its graph rotated around the x-axis.

$\boxed{\text{F}}$ $\sqrt{x}$ on $[0, \pi]$

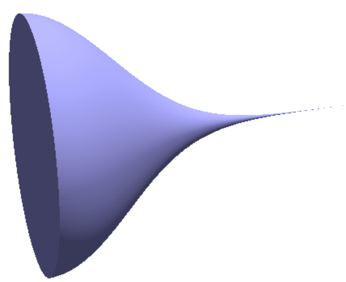


A

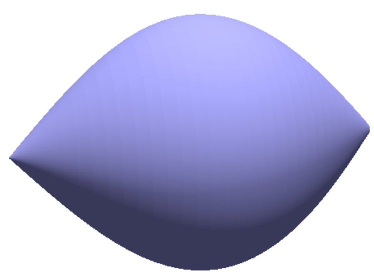
B



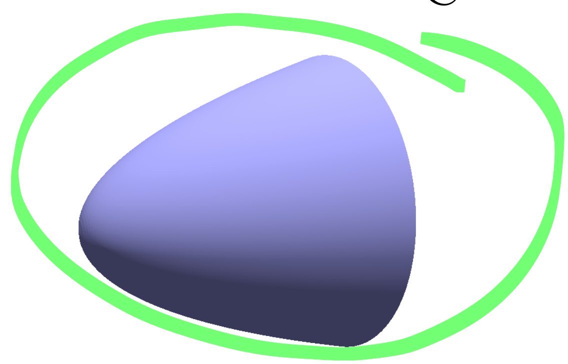
C



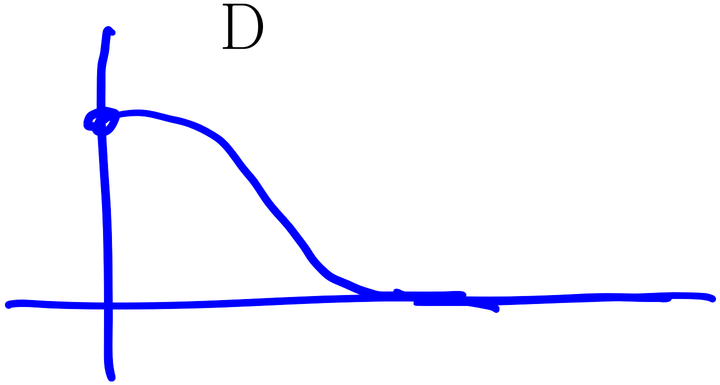
D

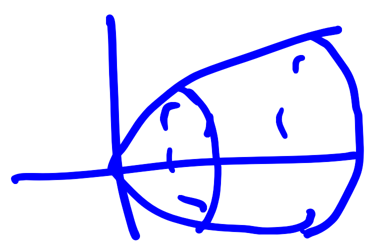


E



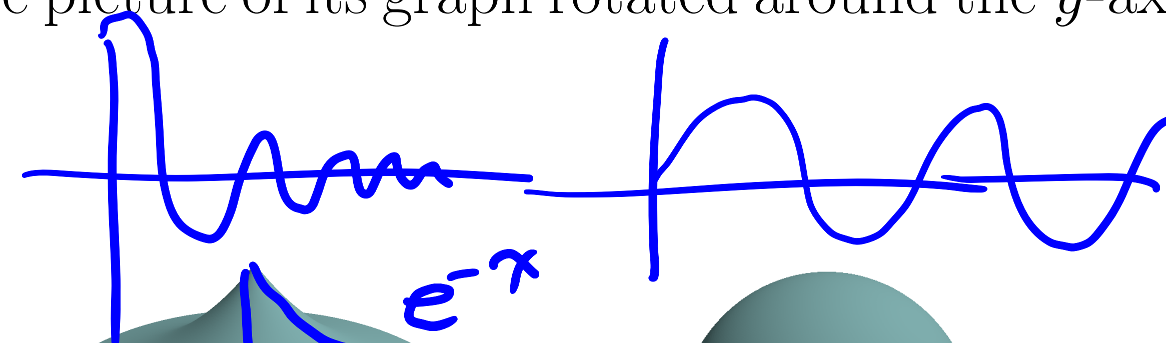
F

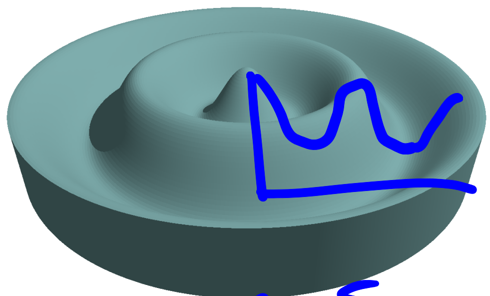




Match each formula with the picture of its graph rotated around the y-axis.

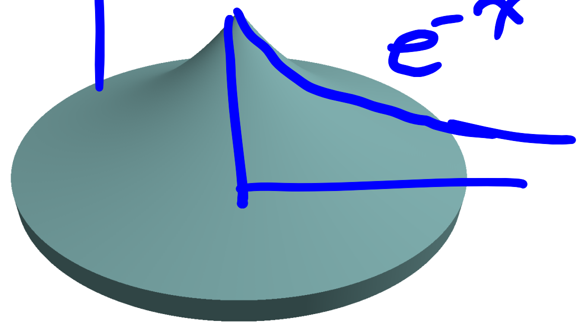
$\boxed{F}$ $\sin(x)/x$ on $[0, 20]$





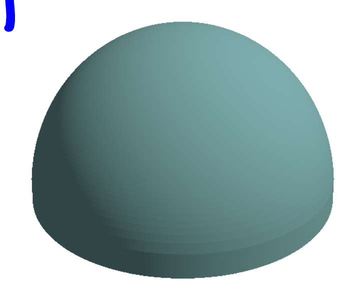
cos

A

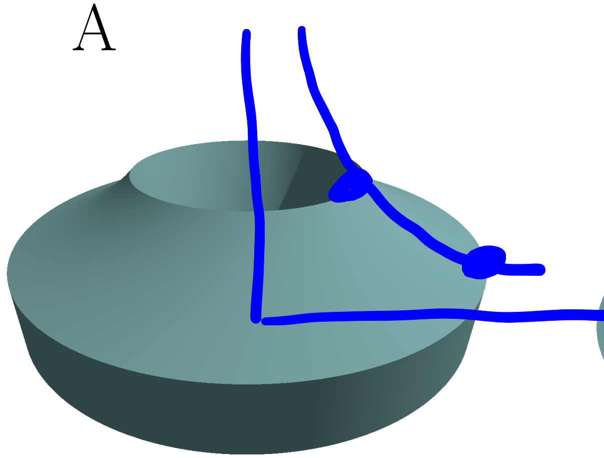


e^{-x}

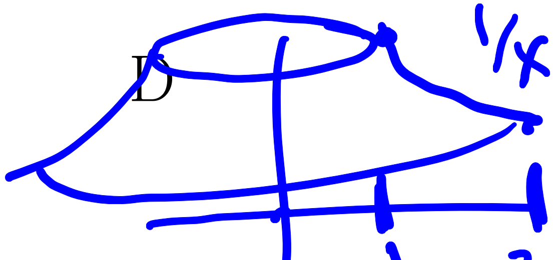
B



C



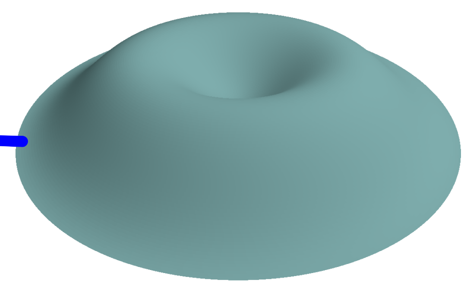
D



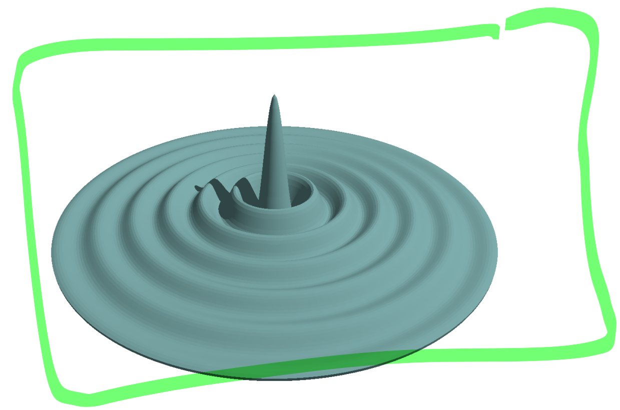
1/x

1

2



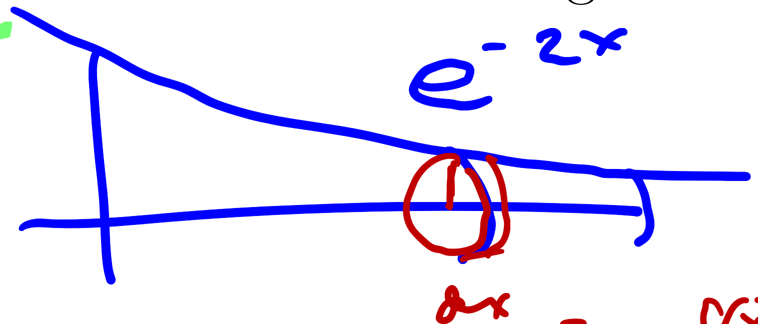
E



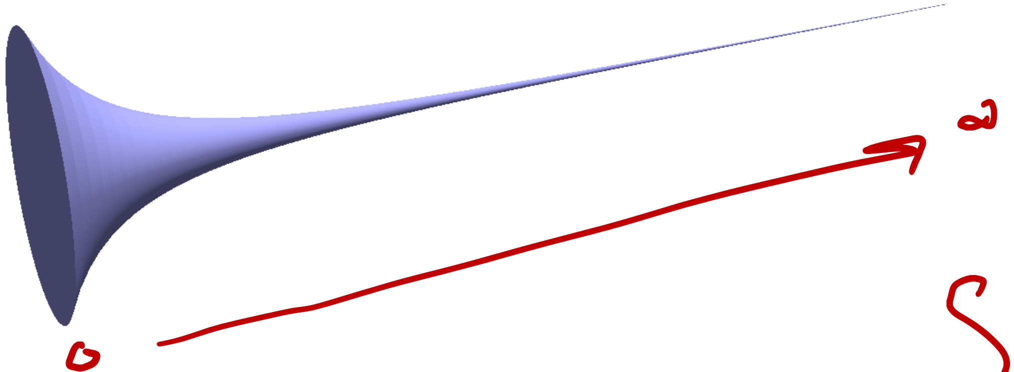
F

Page 4 Quiz 5

The region below is $\{0 \le y \le e^{-2x} : 0 \le x < \infty\}$, rotated around the x-axis. What is the integral formula for the volume using the disk method?



$A = \pi r^2 = \pi f(x)^2$



$$\int_0^\infty \pi (e^{-2x})^2 \, dx$$

$$\boxed{= \pi \int_0^\infty e^{-4x} \, dx}$$

$u = -4x \quad du = -4\,dx$

$$= \frac{\pi}{-4} \int (-4) e^{-4x} \, dx$$

$$= \frac{-\pi}{4} \int e^u = \frac{-\pi}{4} e^{-4x}$$

Compute the volume.

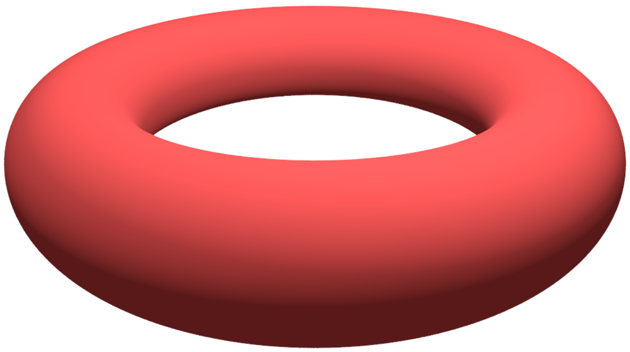
$$-\frac{\pi}{4} e^{-4x} \Big|_0^\infty$$

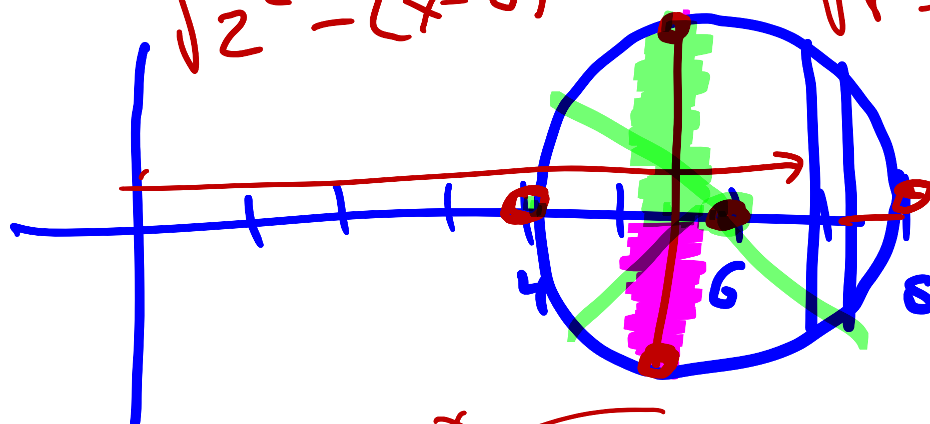
$$-\frac{\pi}{4} [0 - 1]$$

$$= \boxed{\pi/4}$$

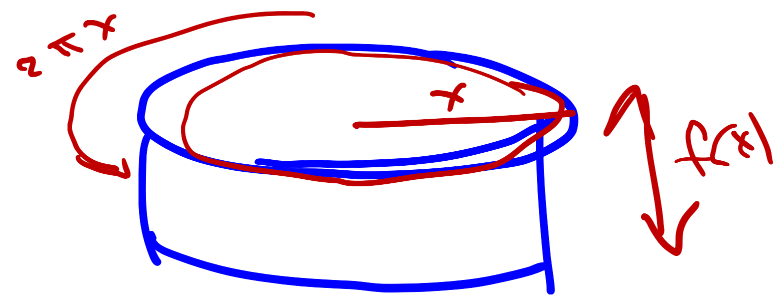
The region below is the disk of radius 2 centered at $x = 6, y = 0$ rotated around the y-axis. Using the method of cylindrical shells, the volume is given by which integral?

$\sqrt{2^2 - (x-6)^2}$ $\quad$ $\sqrt{r^2 - (x-x_0)^2}$





$$2\pi \int_a^b x\, f(x)\, dx$$

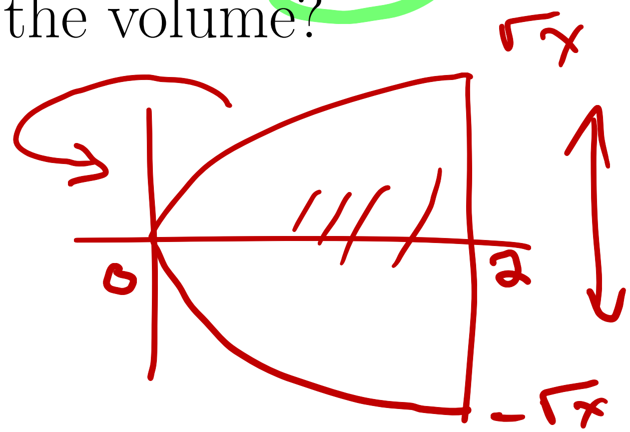


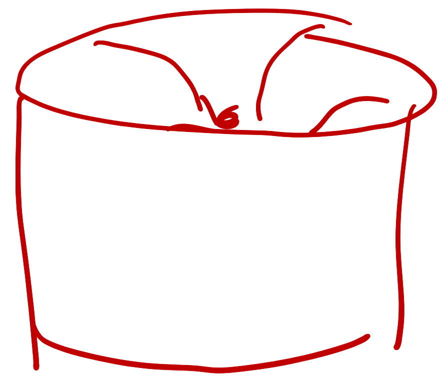
$$2\pi \int_4^8 x\, 2\sqrt{4-(x-6)^2}$$

$$= 4\pi \int_4^8 x \sqrt{4-(x-6)^2}\, dx$$

don't have to evaluate on Mid #2.

The region $\{(x,y) : |y| \le \sqrt{x}, 0 \le x \le 2\}$ is rotated around the y axis. What is the volume?





$$2\pi \int x \, f(x) \, dx$$

$$2\pi \int_0^2 x \cdot 2\sqrt{x} \, dx = 4\pi \int_0^2 x^{3/2} \, dx$$

$$= 4\pi \frac{2}{5} x^{5/2} \Big|_0^2$$

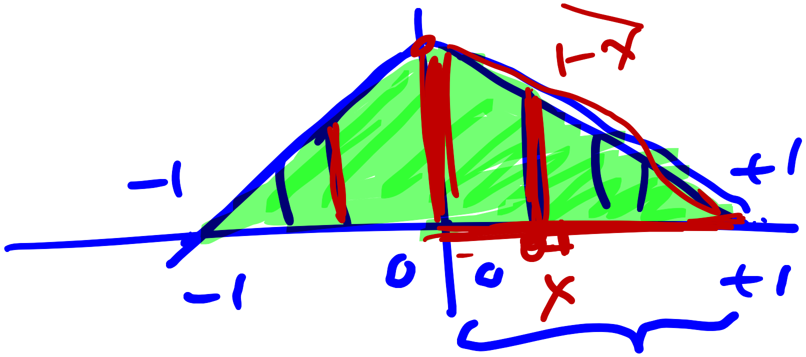
$$2^{5/2} = 2^2 \cdot 2^{1/2} \qquad = 4\pi \frac{2}{5} 2^{5/2}$$

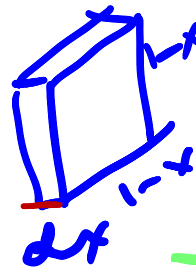
$$= \frac{8\pi}{5} 2^{5/2} = \frac{32\pi}{5} \sqrt{2}$$

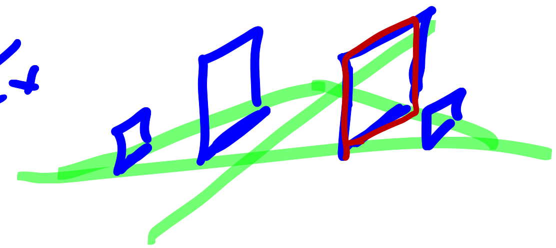
page 5

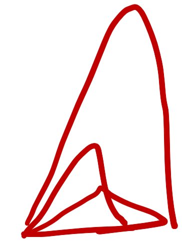
HW

The base of 3 dimensional shape is $\{(x, y) : 0 < y < 1 - |x|\}$ and each cross section of the shape perpendicular to the x-axis is a vertical square with one edge on the xy-plane. What is the volume of this region?









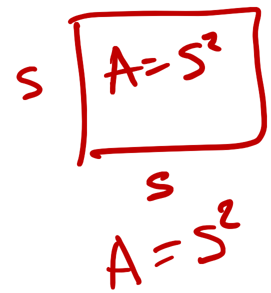
$$2 \cdot \int_0^1 (1-x)^2 \, dx \leftarrow$$

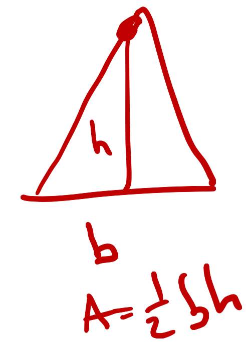
$$2 \int_0^1 1 - 2x + x^2 \, dx$$

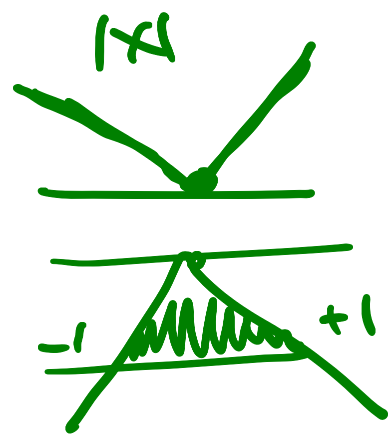
$$2 \left[x - x^2 + \tfrac{1}{3} x^3 \right]_0^1$$

$$= 2 \left[1 - 1 + \tfrac{1}{3} \right] - 0$$

$$= 2/3$$

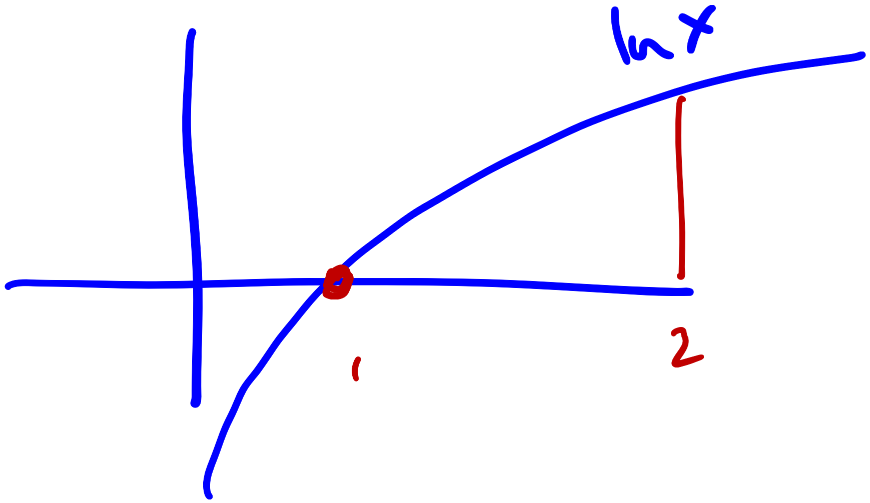






page 5

Which integral gives the arclength of the graph of $y = \ln(x)$ over $[1, 2]$?



$$\int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$f = \ln x$

$f' = 1/x$

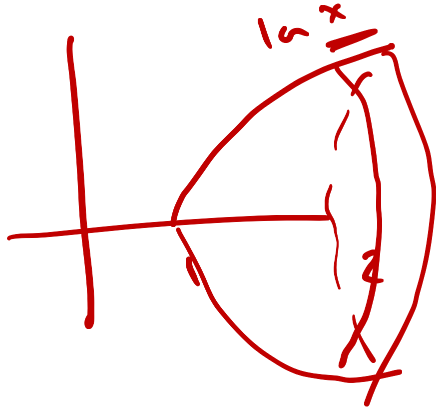
$$\int_1^2 \sqrt{1 + (1/x)^2}\, dx$$

$$\boxed{= \int_1^2 \sqrt{1 + x^{-2}}\, dx}$$

page 5

Which integral gives the area of the graph of $y = \ln(x)$ over $[1, 2]$ rotated around the x-axis?

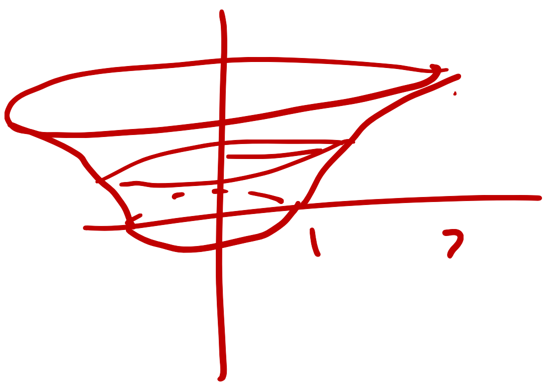
$$\int 2\pi f(x) \sqrt{1+(f'(x))^2}\, dx$$



$$2\pi \int_1^2 \ln(x) \sqrt{1+x^{-2}}\, dx$$

What is integral when we rotate around the y-axis?

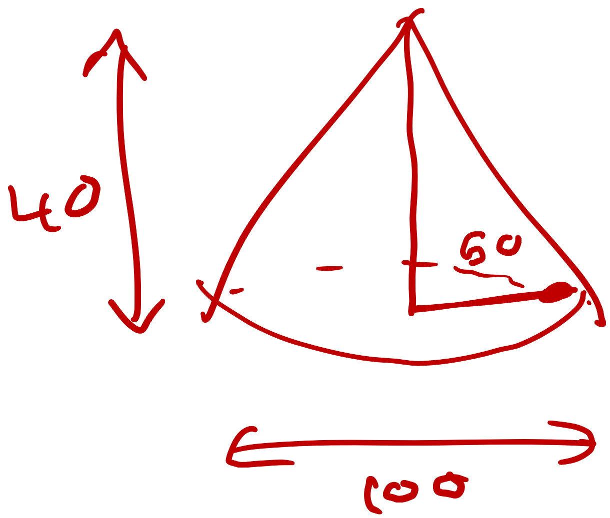
$$\int 2\pi x \sqrt{1+(f'(x))^2}\, dx$$



$$2\pi \int_1^2 x \sqrt{1+x^{-2}}\, dx$$

page 6 Quiz 6

A pile of sand is shaped like a cone 40 feet high and a circular base 100 feet in diameter. What is the volume of the sand pile?

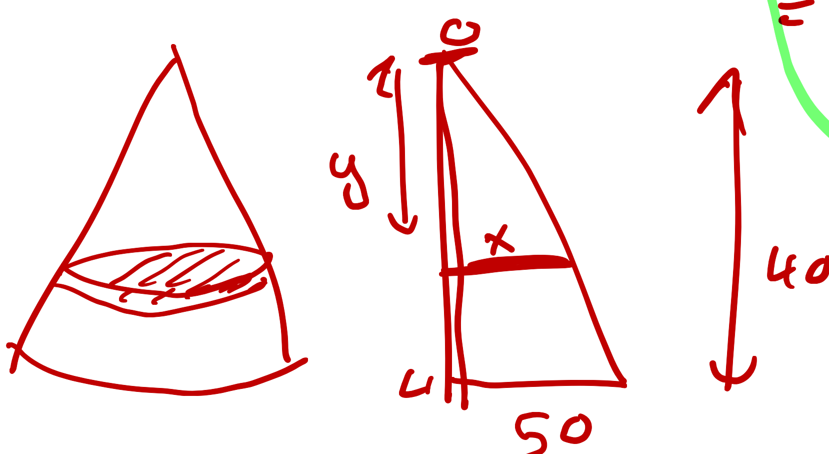


$$V = \frac{1}{3}\underbrace{h}_{\text{height}} \cdot \underbrace{A}_{\text{base area}}$$

$$= \frac{1}{3} \cdot 40 \cdot \pi\, 50^2$$

$$= \frac{1}{3}\, 40\, \pi\, 2500$$

$$\boxed{= \frac{100000}{3}\pi}$$



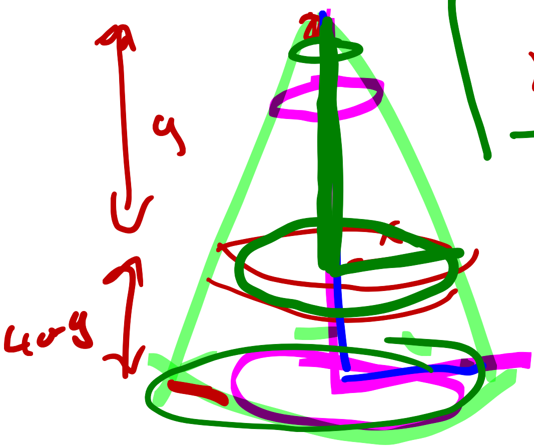
$$\frac{40}{50} = \frac{y}{x}$$

$$x = \frac{50}{40} \cdot y$$

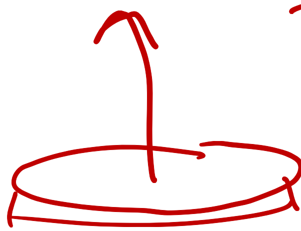
$$\int \pi x^2 = \int_0^{40} \pi \left(\frac{50}{40}\right)^2 y^2\, dy = \pi \frac{50^2}{40^2} \cdot \frac{1}{3} y^3 \Big|_0^{40}$$

$$= \pi \frac{50^2}{\cancel{40^2}} \frac{1}{3}\, 40^{\cancel{3}} = \frac{100000\pi}{3}$$

Suppose sands weights 75 lbs per cubic foot. How much work (in foot-pounds) is done lifting the sand from ground level to build cone?



$$\boxed{x = \frac{50}{40} y}$$



$$A = \pi \left(\frac{50}{40}\right)^2 y^2$$

$$75 \times \int_0^{40} \pi \left(\frac{50}{40}\right)^2 y^2 (40-y)\, dy$$

$$= \pi \frac{50^2}{40^2} \int_0^{40} 40y^2 - y^3 \quad \times 75$$

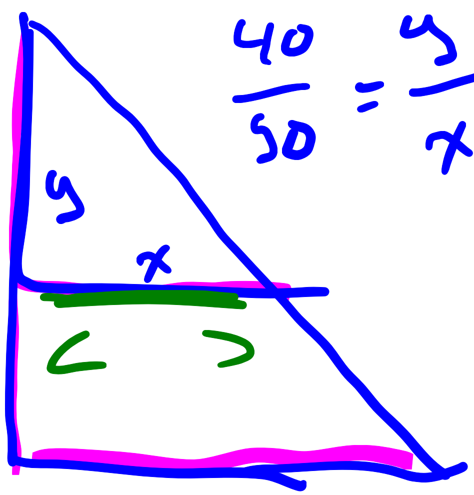
$$= \pi \frac{50^2}{40^2} \left[\frac{40}{3} y^3 - \frac{1}{4} y^4\right]_0^{40} \quad ''$$

$$= \pi \frac{50^2}{40^2} \left[\frac{40^4}{3} - \frac{1}{4} 40^4\right] \quad ''$$

$$= \pi \frac{50^2}{\cancel{40^2}} \frac{\cancel{40^4}\, 40^2}{12} \quad ''$$

$$= \pi \frac{50^2 \cdot 40^2}{12} = \frac{(2000)^2}{12} \pi \quad ''$$

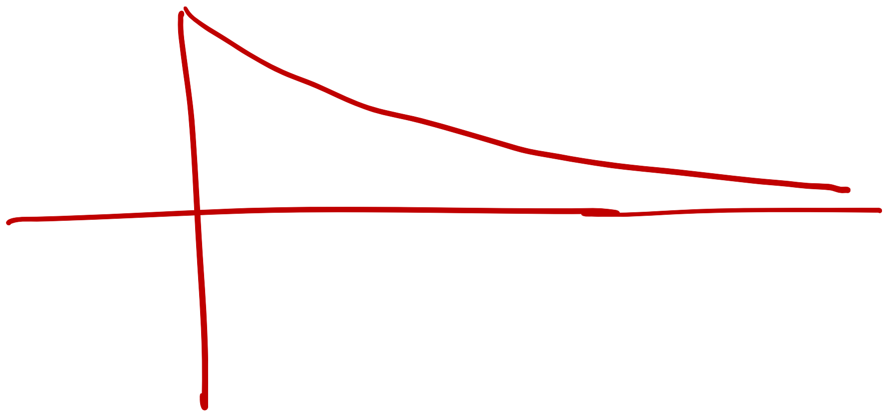
$$= 75 \cdot \frac{4000000}{12} \pi$$



$$\boxed{\frac{40}{50} = \frac{y}{x}}$$

"similar triangles"

The population density of a town is estimated to be $20000e^{-x/2}$ people per square mile, where x is the distance in miles from the city center. Which integral gives the number of people living within 10 miles of the city center?



$$(xe^{-x/2})' = e^{-x/2} + \underbrace{xe^{-x/2}}(-\tfrac{1}{2})$$

$$\left[(-2)xe^{-x/2} + 4e^{-x/2}\right]'$$

$$= \cancel{(-2)}e^{-x/2} + \underline{xe^{-x/2}} \cancel{-2e^{-x/2}}$$

$$\text{mass} = \int_0^r \rho(x)\cdot 2\pi x\,dx$$

$$= \int_0^{10} (20000)e^{-x/2}\cdot 2\pi x\,dx$$

$$= 40000\pi \int_0^{10} \underbrace{xe^{-x/2}}\,dx$$

"integration by parts"