MAT 126.01, Prof. Bishop, Thursday, Nov 5, 2020
Section 3.7: Improper integrals
Quiz 9 review

An integral is **improper** if either the interval is unbounded or the function is unbounded.

$$\int_0^\infty e^{-x}dx, \qquad \int_{-\infty}^\infty \frac{dx}{1+x^2}$$

$$\int_0^1 \frac{dx}{x^2} \qquad \int_0^{\pi/2} \tan(x)dx$$

Integrals over $[a, \infty)$:

$$\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

We say integral **converges** if limits exists and is finite. Otherwise the integral **diverges**.

Evaluate $\int_0^\infty e^{-x}dx$.

Evaluate $\int_0^\infty \frac{dx}{1+x^2}$

Evaluate $\int_1^\infty \frac{dx}{x}$

Find volume of $1/x$ rotated around x-axis for $1 \leq x < \infty$.

Find surface of $1/x$ rotated around x-axis for $1 \leq x < \infty$.

Integrals over $(-\infty, b)$:

$$\int_{-\infty}^{b} f(x)dx = \lim_{t \to -\infty} \int_{-t}^{b} f(x)dx$$

Integrals over $(-\infty, \infty)$:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)dx$$

If f is continuous on $(a, b]$ but unbounded as $x \to a$, define

$$\int_a^b f(x)dx = \lim_{t \searrow a} \int_t^b f(x)dx.$$

Evaluate $\int_0^1 \frac{dx}{\sqrt{x}}$

Evaluate $\int_0^1 \frac{dx}{\sqrt{x}}$

Show $\int_0^1 x^p dx$ diverges if and only if $p < -1$.

Show $\int_1^\infty x^p dx$ diverges if and only if $p > -1$.

Suppose $0 \leq f(x) \leq g(x)$ on $[a, \infty]$.

If $\int_a^\infty f(x)dx$ diverges, then $\int_a^\infty g(x)dx$ diverges.

If $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ converges.

Even when we can't evaluate exactly, we can often decide if an integral diverges or converges by comparing to a known one.

Does $\int_1^\infty x^{-2} \sin(x^3) dx$ converge or diverge?

Does $\int_1^\infty x^{10}e^{-x}dx$ converge or diverge?

Does $\int_0^1 \frac{1}{x} \ln x dx$ converge or diverge?

Quiz 9 review: Sections 3.3, 3.4 and 3.7.

Page 1:

- 4 integrals: converge or diverge
- 1 problem: choose correct reference triangle
- 2 integrals: choose correct trig substitution

Page 2:

- choose correct formula from rational function graph
- find partial fraction expansion (non-repeated linear terms)
- long division of polynomials

Write C if the improper integral converges or a D if it diverges (is infinite or undefined).

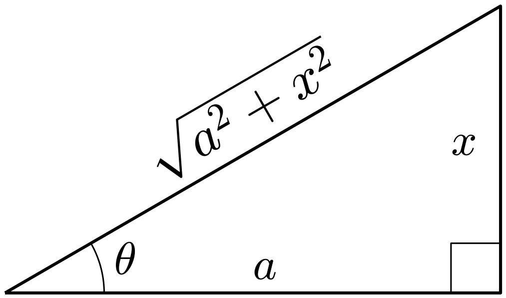
☐ $\int_0^1 \frac{dx}{x^2}$

☐ $\int_0^\infty x^8 e^{-8x} dx$

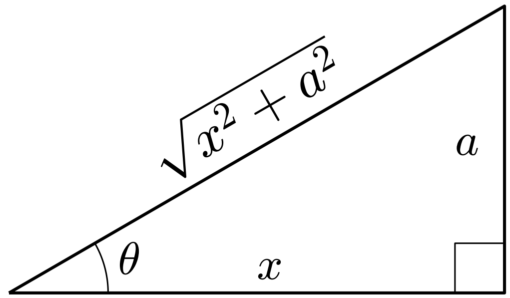
☐ $\int_0^\infty \frac{x^7+x^4+1}{x^8+3x^4-x^2+5} dx$

☐ $\int_{-\infty}^\infty \sin(x) e^{-x} dx$

☐ Which reference triangle corresponds to $\cot\theta = \frac{x}{a}$?



A



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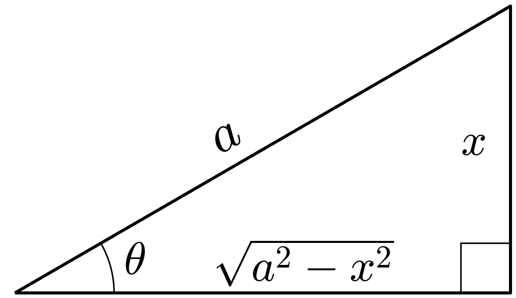

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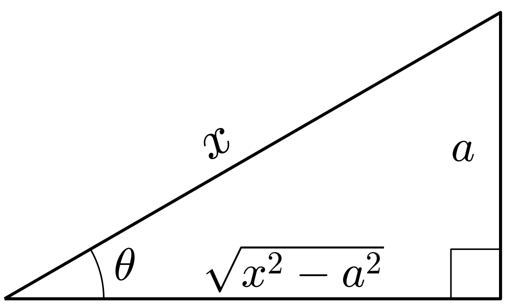

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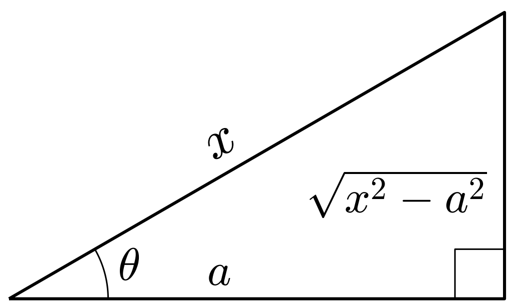

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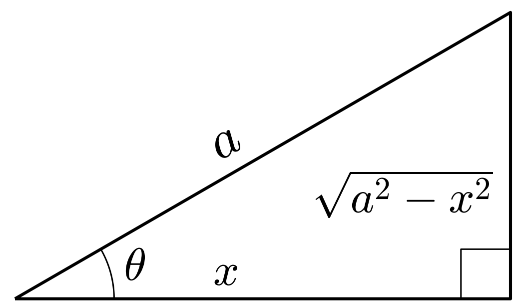


F

For each integral choose the appropriate substitution from the right.

☐ $\int \frac{dx}{(1-x^2)^{3/2}}$

☐ $\int \sqrt{x^2+16}dx$

(a) $x = \tan\theta$
(b) $x = \sec\theta$
(c) $x = \sin\theta$
(d) $x = 2\tan\theta$
(e) $x = 2\sec\theta$
(f) $x = 2\sin\theta$
(g) $x = 3\tan\theta$
(h) $x = 3\sec\theta$
(i) $x = 3\sin\theta$
(j) $x = 4\tan\theta$
(k) $x = 4\sec\theta$
(l) $x = 4\sin\theta$

Which is the partial fraction expansion for the graph below?

(a) $\frac{A}{x-4} + \frac{Bx+C}{(x+4)^2} + \frac{D}{1+(x+1)^2}$

(b) $\frac{A}{x+3} + \frac{Bx+C}{x^2} + \frac{D}{1+(x-3)^2}$

(c) $\frac{A}{x-2} + \frac{Bx+C}{(x+4)^2} + \frac{D}{1+(x+1)^2}$

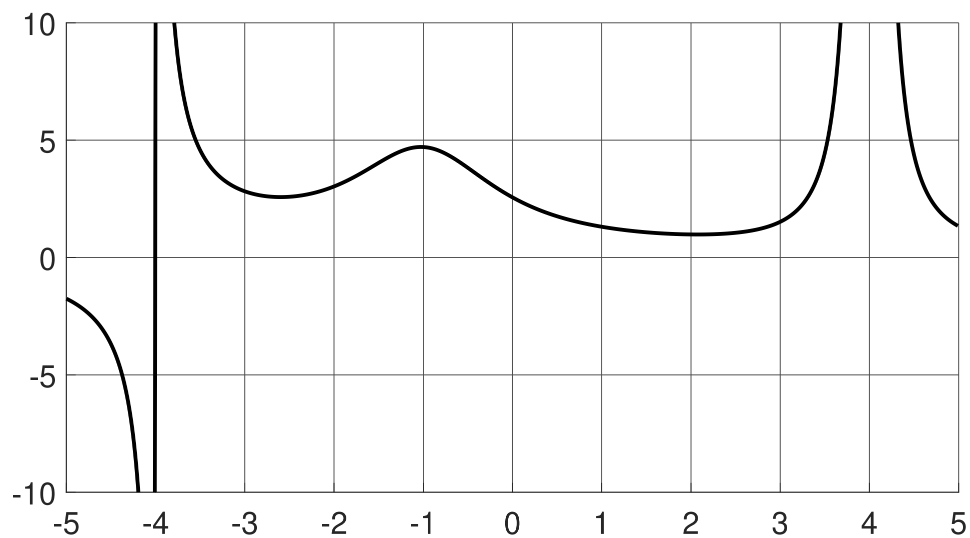
(d) $\frac{A}{x+2} + \frac{Bx+C}{(x-1)^2} + \frac{D}{1+(x-4)^2}$

(e) $\frac{A}{x+4} + \frac{Bx+C}{(x-4)^2} + \frac{D}{1+(x+1)^2}$

(f) $\frac{A}{x+4} + \frac{Bx+C}{(x-3)^2} + \frac{D}{1+x^2}$

(g) $\frac{A}{x+5} + \frac{Bx+C}{(x+2)^2} + \frac{D}{1+(x-4)^2}$

(h) none of these



Find A, B where

$$\frac{9x-1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}.$$

Simplify using long division of polynomials:

$$\frac{4x^3 - x^2 + x}{x^2 - 1}.$$