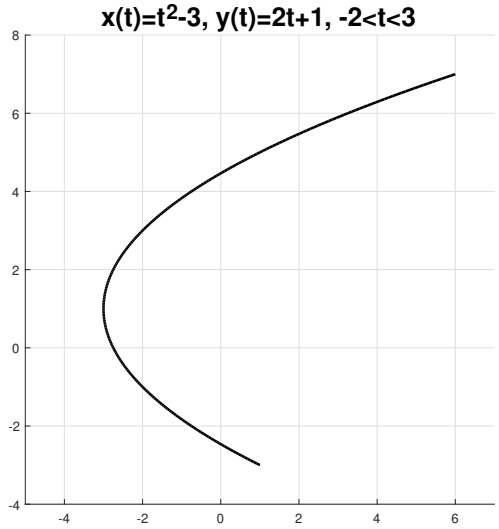
MAT 126.01, Prof. Bishop, Thursday, Nov 19, 2020
Section 7.2 Calculus of Parametric Curves

- Find derivatives and tangents to parametric curves
- Find area under a parametric curve.
- Arclength of parametric curve.
- Area of rotated parametric curve.

Derivatives of parametric equations:

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}.$$

Example: calcuate derivative at time t of $x(t) = t^2 - 3$, $y(t) = 2t - 1$.



What is tangent line at $t = 2$?

Second derivatives:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}}.$$

Find second derivative of

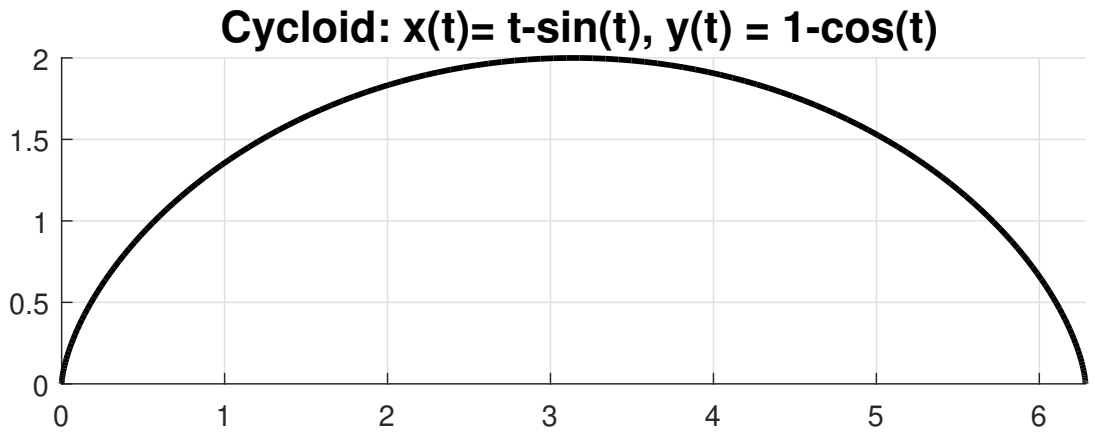
$$x(t) = t^2 - 3, \qquad y(t) = 2t - 1.$$

Area under a parametric curve:

$$A = \int_a^b y(t)x'(t)dt.$$

Area under a cycloid.

$$x(t) = t - \sin(t), \qquad y(t) = 1 - \cos(t).$$



What is the area of the ellipse $\frac{1}{4}x^2 + y^2 \leq 1$?

Arclength of a parametric curve:

$$\text{length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}.$$

What is length of circle $x(t) = 2\cos(t), \quad y(t) = 2\sin(t)$.

Surface area of revolution

$$\text{area} = 2\pi \int_a^b y(t)\sqrt{(x'(t))^2 + (y'(t))^2}$$

Find surface area of a sphere of radius r.