MAT 126.01, Prof. Bishop, Tuesday, Nov 17, 2020
Last minute questions on Midterm 3
Section 7.1 Parametric equations

In a usual function the y coordiate is given as a function of x

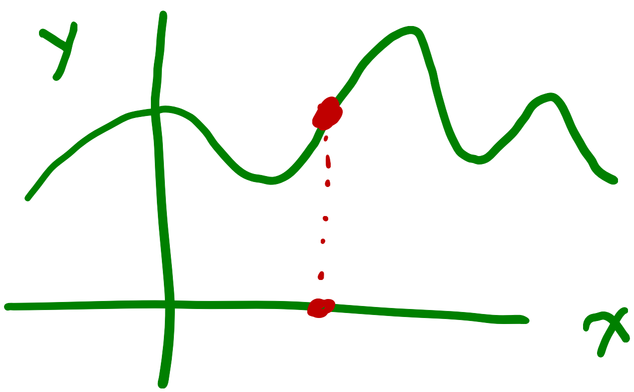
$$y = f(x).$$

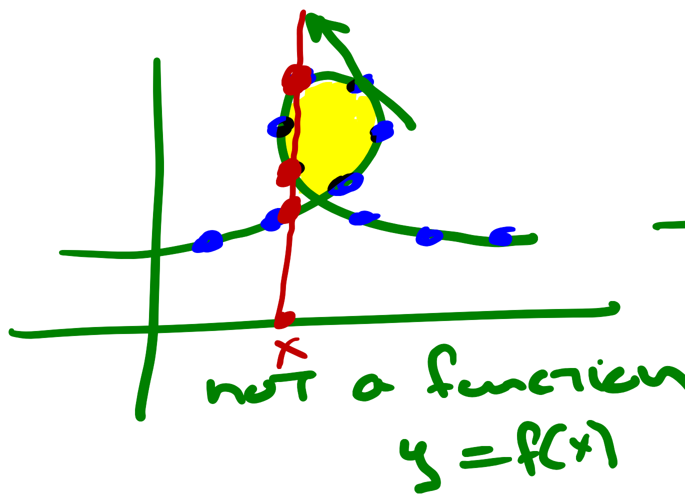
In a parametric equation the x and y coordinates are both given as functions of a third parameter t

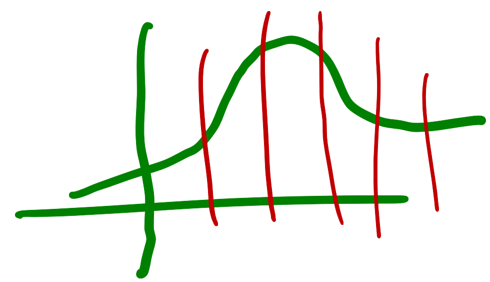
$$(x(t), y(t))$$

If $x(t) = t$, the two ideas are the same.

But in general a parametric equation describes curves that are not graphs of functions.





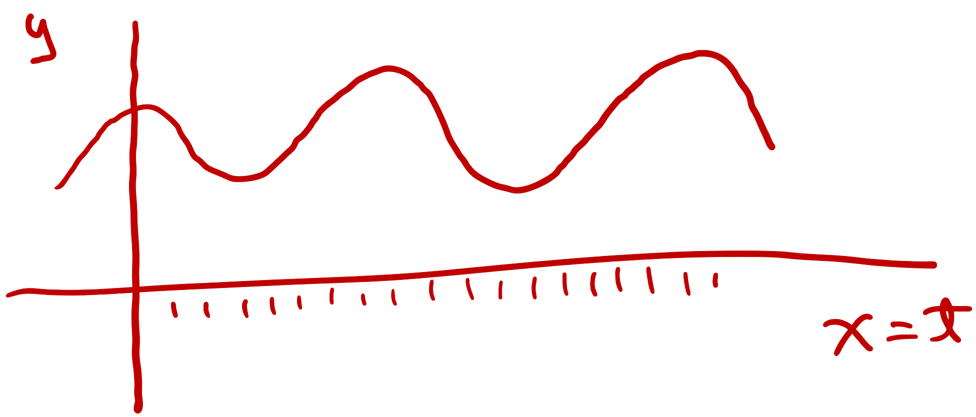


Easiest case is when $x(t) = t$. Then plot of $(x(t), y(t))$ is just graph of $y(t)$.

$y = f(x)$ $\qquad\qquad (t, f(t))$

$y(t) = f(t)$

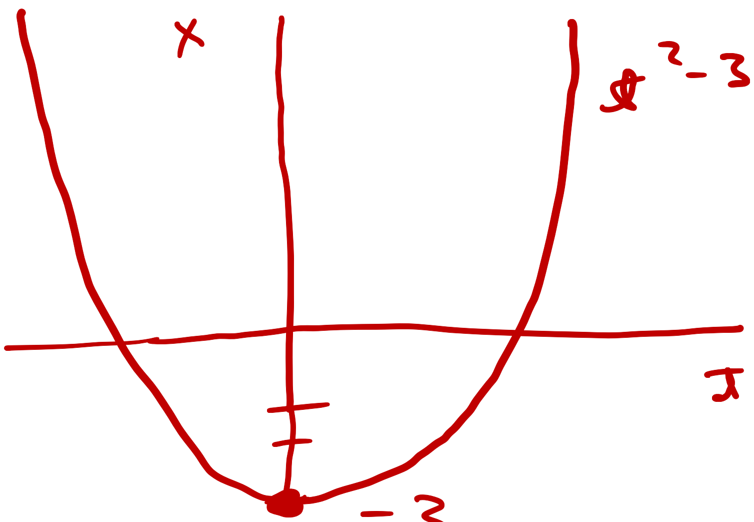
$x(t) = t$

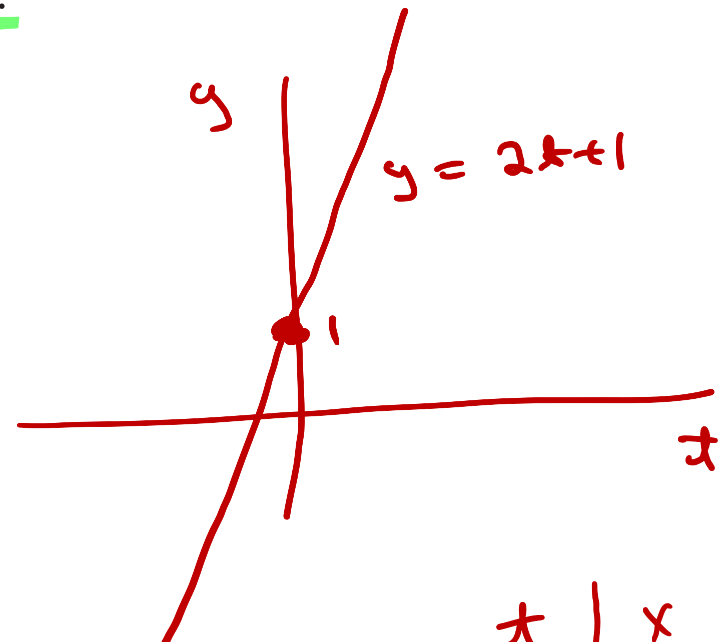


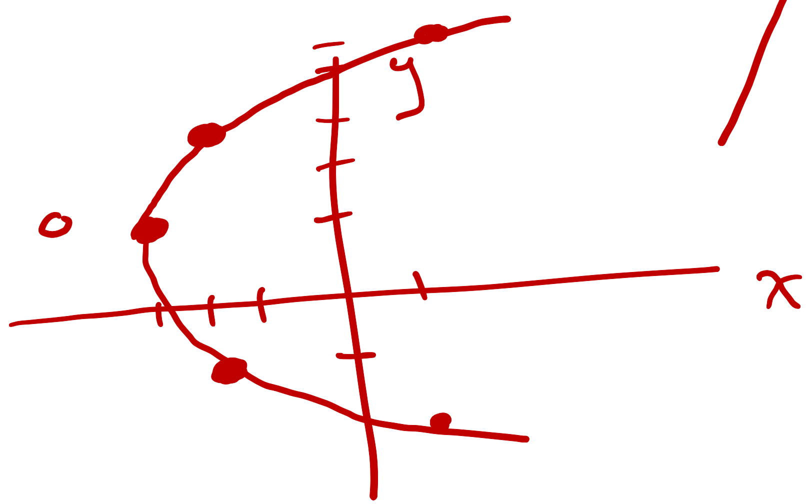
Eliminating the parameter.

Idea is to write the two equations $x = x(t)$ and $y = y(t)$ as one equation involving x and y.

Example: $x(t) = t^2 - 3$, $y(t) = 2t + 1$.

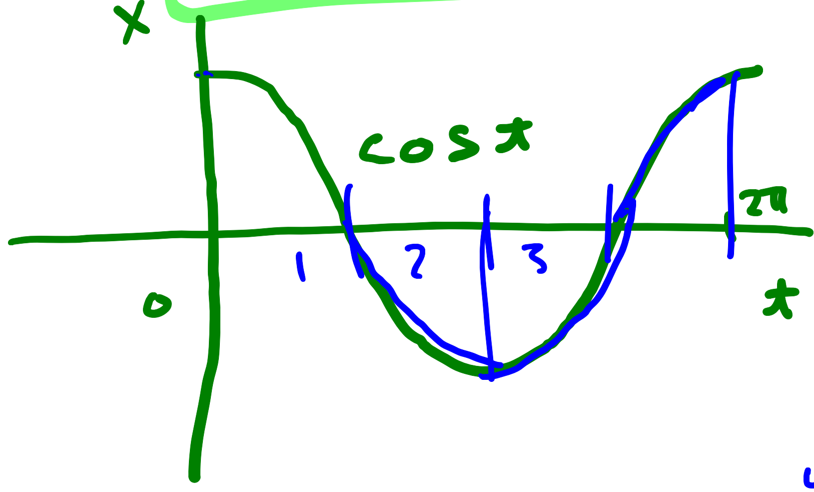


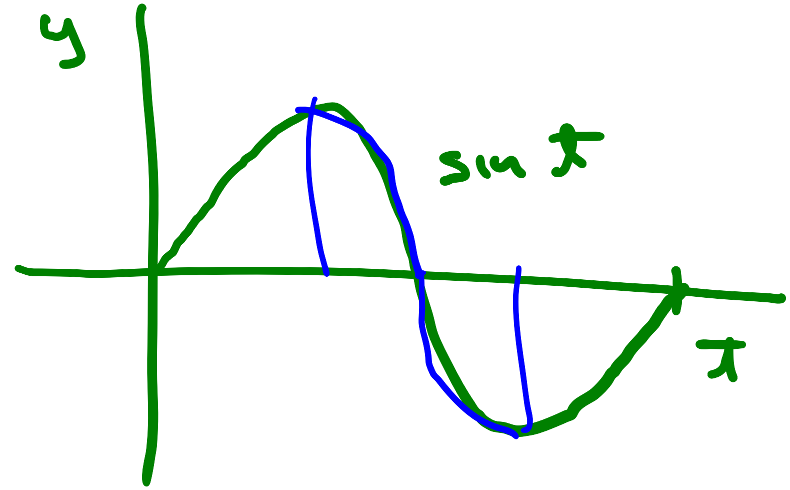


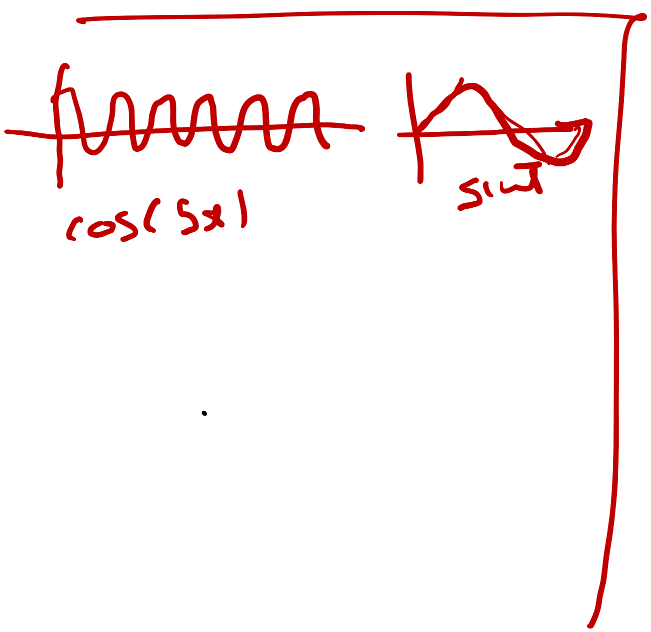


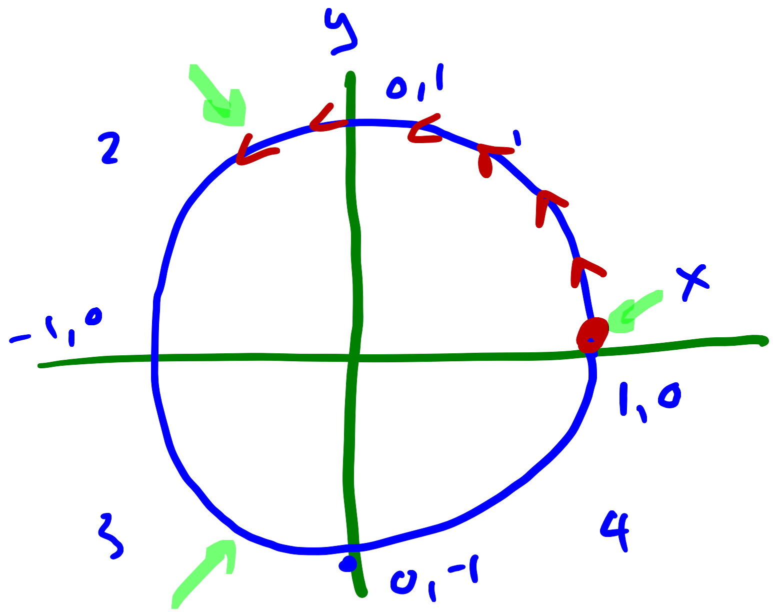
t	x	y
0	-3	1
1	-2	3
2	1	5
-1	-2	-1
-2	1	-3

Example: $x(t) = \cos(t)$, $y(t) = \sin(t)$.







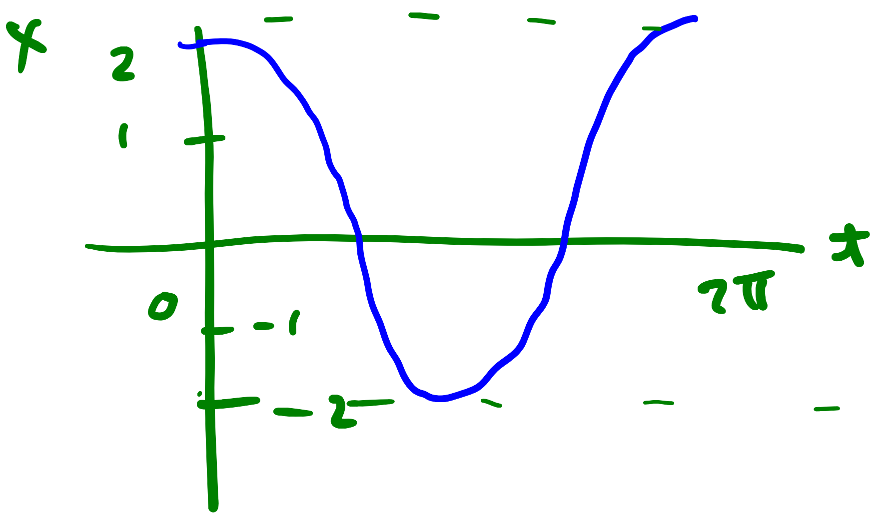


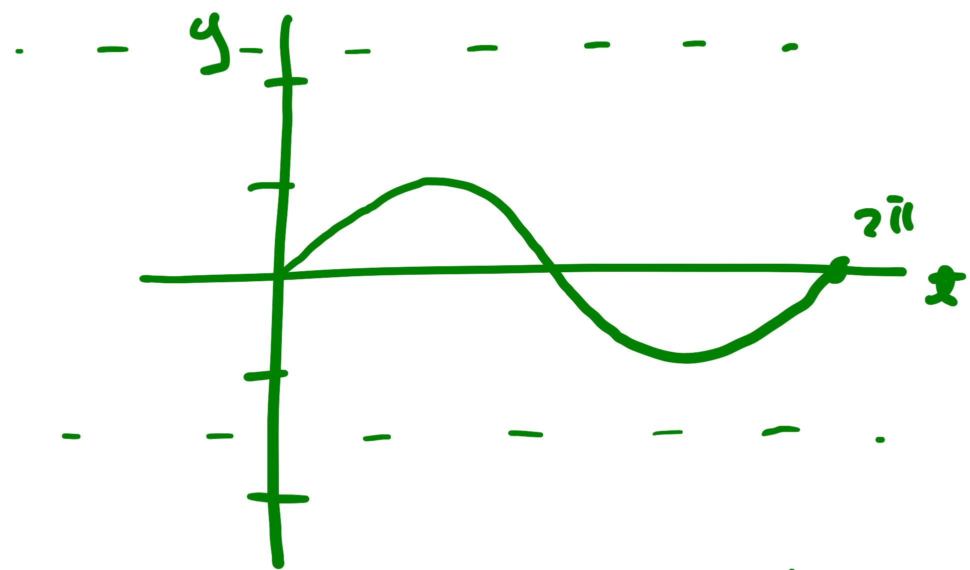
$x^2 + y^2 = \cos^2 + \sin^2$

$= 1$

$\boxed{x^2 + y^2 = 1}$

Example: Find equation for $x(t) = \boxed{2\cos(t)}$, $y(t) = \sin(t)$. What kind of shape is this?





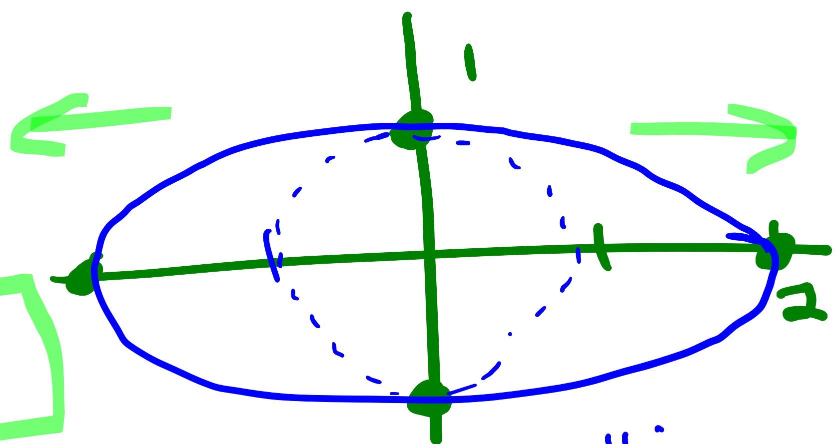
$x = 2\cos$

$\frac{x}{2} = \cos$

$y = \sin$

$$\boxed{\left(\frac{x}{2}\right)^2 + y^2 = 1}$$

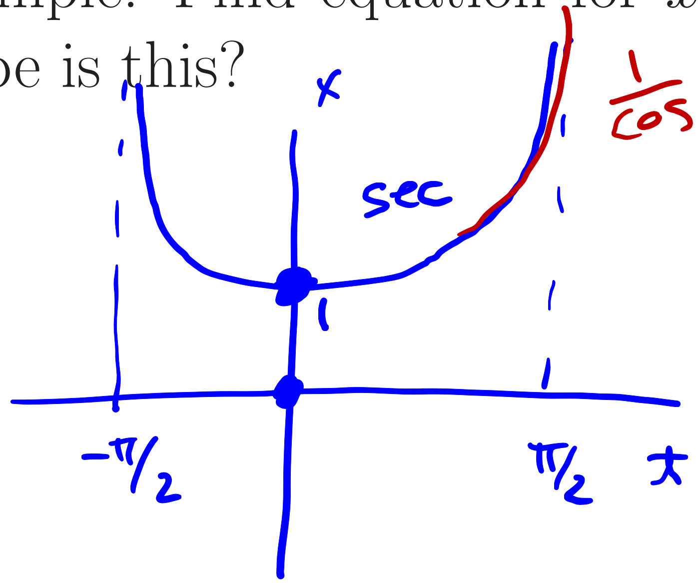
$\cos^2 + \sin^2$

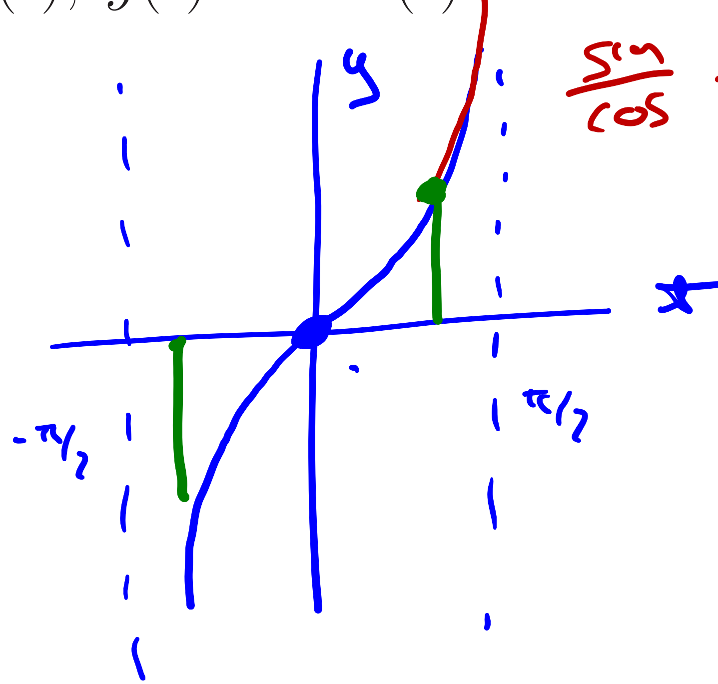


ellipse

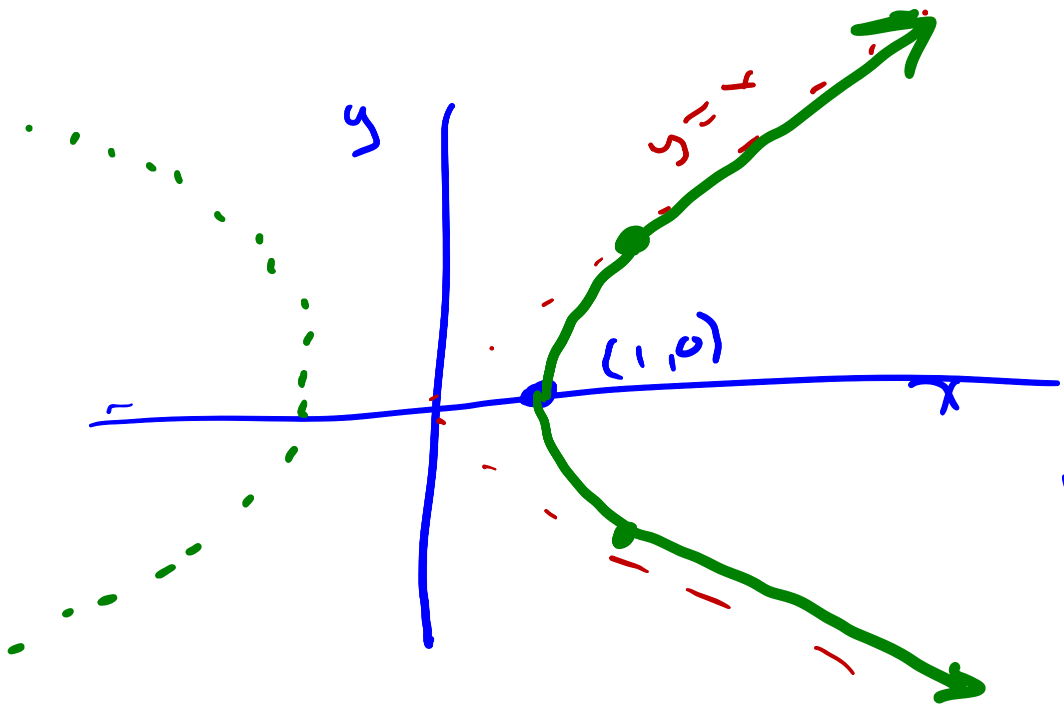
t	x	y
0	2	0
$\pi/2$	0	1
π	-2	0
$\frac{3\pi}{2}$	-1	0

Example: Find equation for $x(t) = \sec(t)$, $y(t) = \tan(t)$. What kind of shape is this?





$\frac{\sin}{\cos} \approx \frac{1}{\cos}$



$\boxed{x = \sec \qquad y = \tan}$

$\frac{\cos^2}{\cos^2} + \frac{\sin^2}{\cos^2} = \frac{1}{\cos^2}$

$1 + \tan^2 = \sec^2$

$1 = \sec^2 - \tan^2$

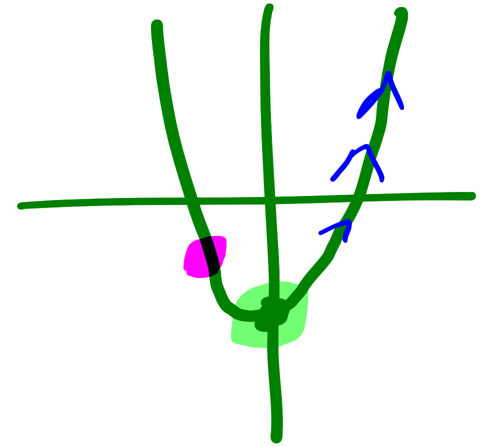
hyperbola

$\boxed{1 = x^2 - y^2}$

Find a parametrization of $y = 2x^2 - 3$.

$x(t) = t$

$y(t) = 2t^2 - 3$



Find a different parametrization of $y = 2x^2 - 3$.

$\boxed{x(t) = 2t - 1}$

$x = 2t - 1$

$x + 1 = 2t$

$\frac{x+1}{2} = t$

$y(t) = 2x^2 - 3$

$= 2(2t-1)^2 - 3$

$= 2(4t^2 - 4t + 1) - 3$

$\boxed{= 8t^2 - 8t - 1}$

What curve does a point on a rolling wheel follow? Called a cycloid.

Assume radius is a.

Assume wheel takes time 2π to make one rotation (makes equation easier).

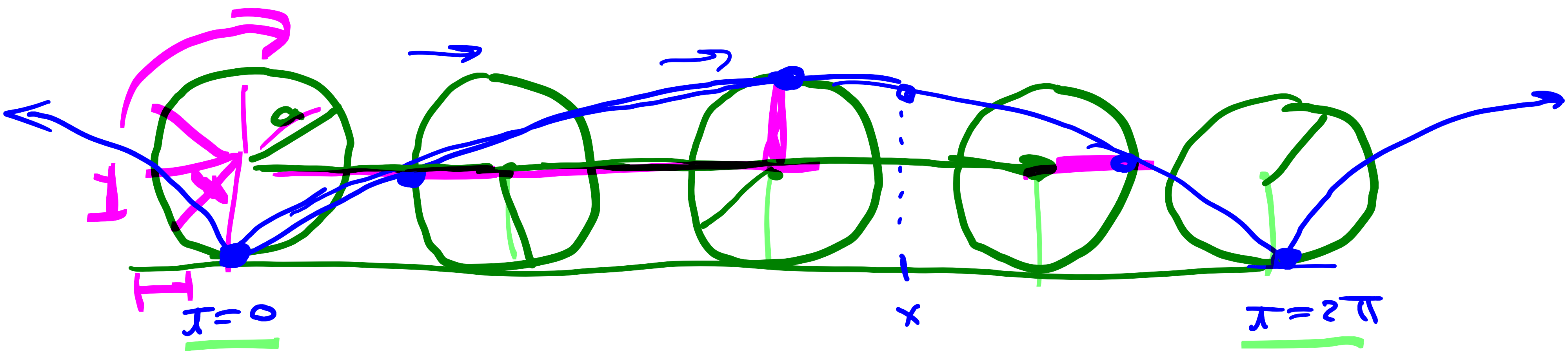
Then center moves by $\boxed{x(t) = at}$, $\boxed{y(t) = a}$.

$$\frac{\text{dist}}{\text{Time}} = \frac{2\pi a}{2\pi} = a$$

Point on bottom of wheel moves by

$$x(t) = at + a\sin(-t) = at - a\sin t = \boxed{a(t - \sin t)},$$

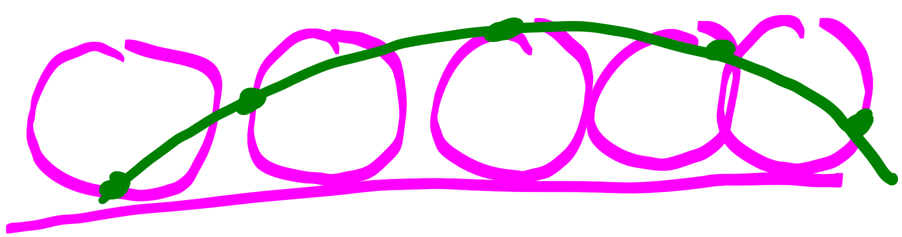
$$y(t) = a - a\cos(-t) = \boxed{a(1 - \cos t)},$$

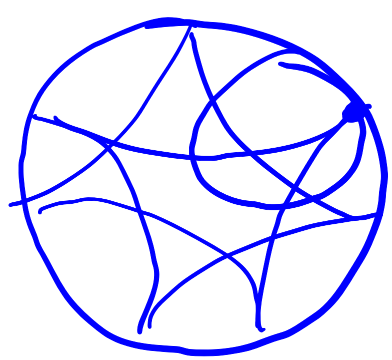


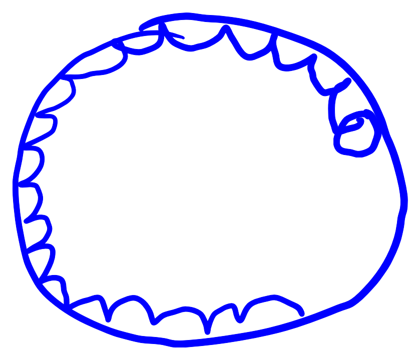
A wheel of radius b rolling inside a circle of radius a:

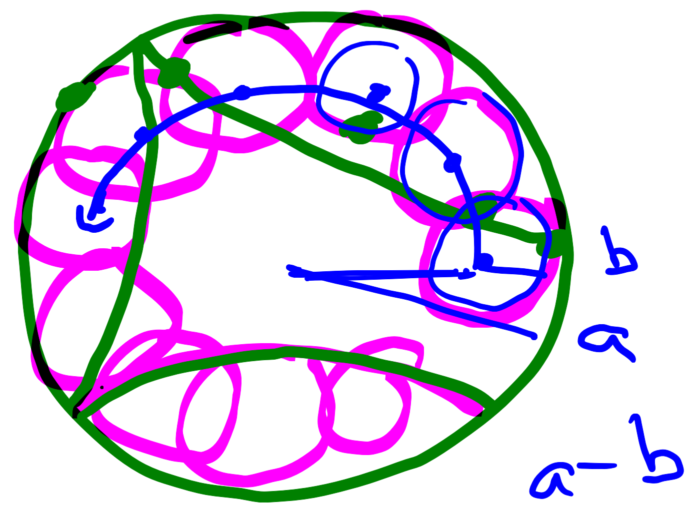
$$x(t) = (a-b)\cos t + b\cos(\frac{a-b}{b}t),$$

$$y(t) = (a-b)\sin t + b\sin(\frac{a-b}{b}t),$$





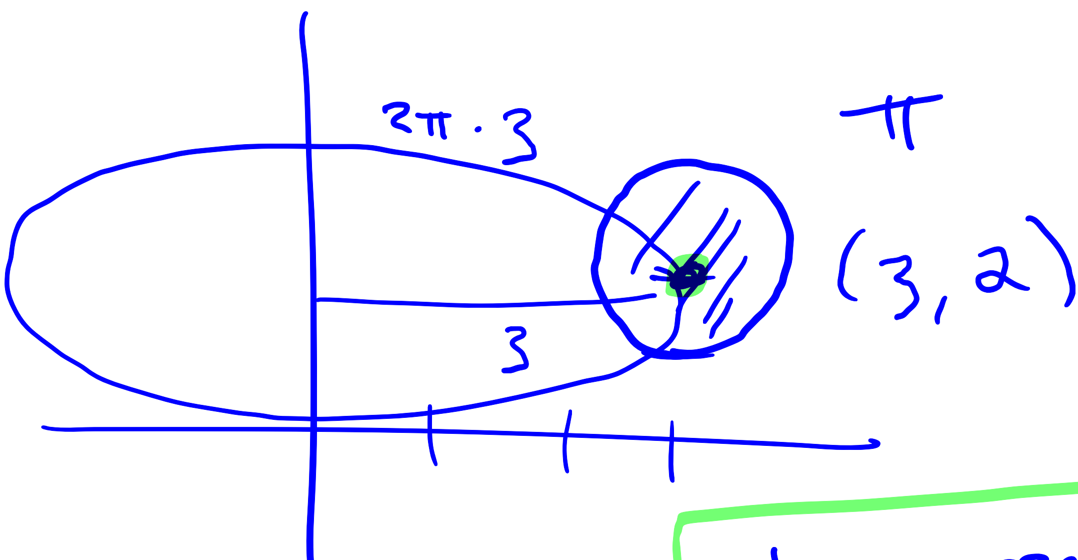




MAT 126 Office Hours

Tue Nov 17

≈ 11:05 – 11:55



radius 1

$\boxed{\text{vol} = \text{area} \times \text{dist.}}$

$= \pi \cdot 6\pi = 6\pi^2$

$$\boxed{\int_2^\infty \frac{1}{x \ln^4 x}\,dx}$$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$\boxed{\int \frac{1}{x \ln^4 x}\,dx} = \int \frac{1}{u^4}\,du = \int u^{-4}\,du$$

$$= \frac{1}{-3} u^{-3}$$

$$\boxed{= \frac{-1}{3} \frac{1}{\ln^3 x}}$$

$$\int_2^\infty = \lim_{t\to\infty} \left[-\frac{1}{3} \frac{1}{\ln^3 x} \right]_2^t$$

$$= \lim_{t\to\infty} \left(-\frac{1}{3} \frac{1}{\ln^3 t} \right) - \left(-\frac{1}{3} \frac{1}{\ln^3 2} \right)$$

$$= 0 + \boxed{\frac{1}{3} \frac{1}{(\ln 2)^3}}$$

$$\int_0^\infty x e^{-x}\,dx$$

$$\int \underbrace{x}_{u}\,\underbrace{e^{-x}\,dx}_{dv} = -xe^{-x} - \int (-e^{-x})\,dx$$

$du = 1\,dx \quad v = -e^{-x}$

$$= -xe^{-x} + \int e^{-x}\,dx$$

$$= \boxed{-xe^{-x} - e^{-x}}$$

$$\int_0^\infty x e^{-x}\,dx = \lim_{t\to\infty} \int_0^t$$

$$= \underbrace{\left[-te^{-t} - e^{-t}\right]}_{\to 0} - \underbrace{\left[0e^0 - e^0\right]}_{0}$$

$$= 0 - -1 = \boxed{1}$$