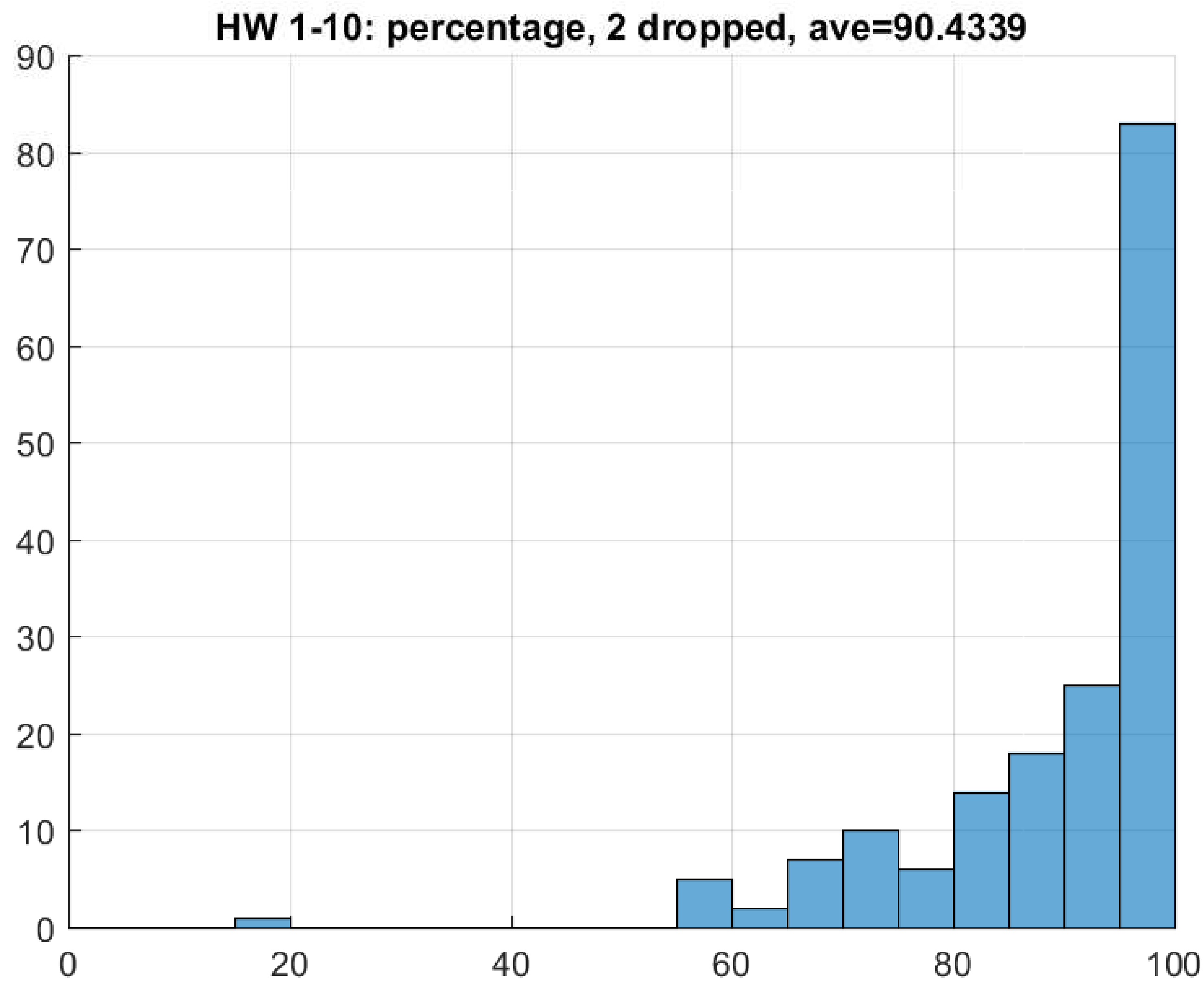
MAT 126.01, Prof. Bishop, Thursday, 12, 2020
Midterm 3 review


HW 1-10: percentage, 2 dropped, ave=90.4339
90
80
70
60
50
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30
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0
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80
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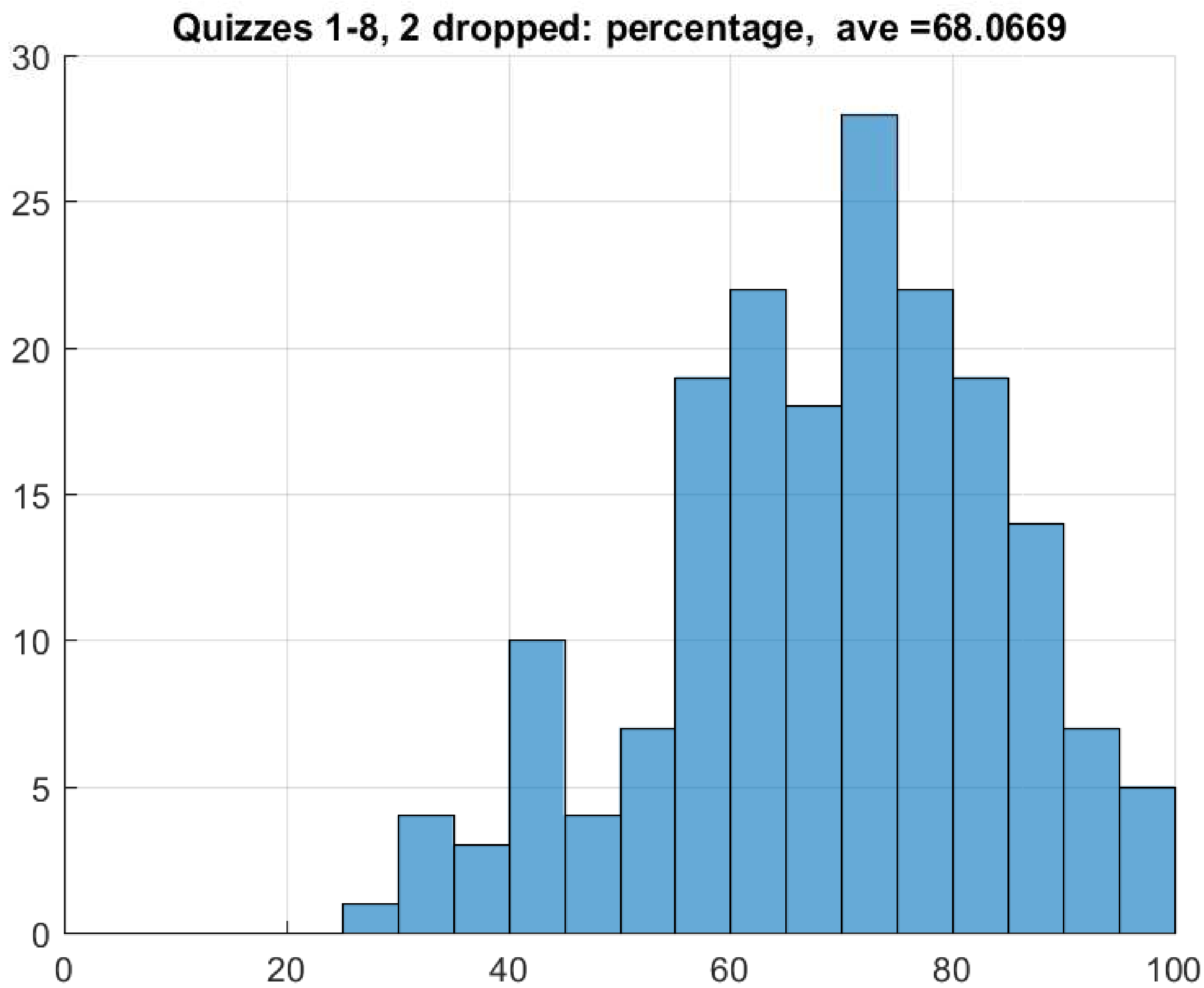
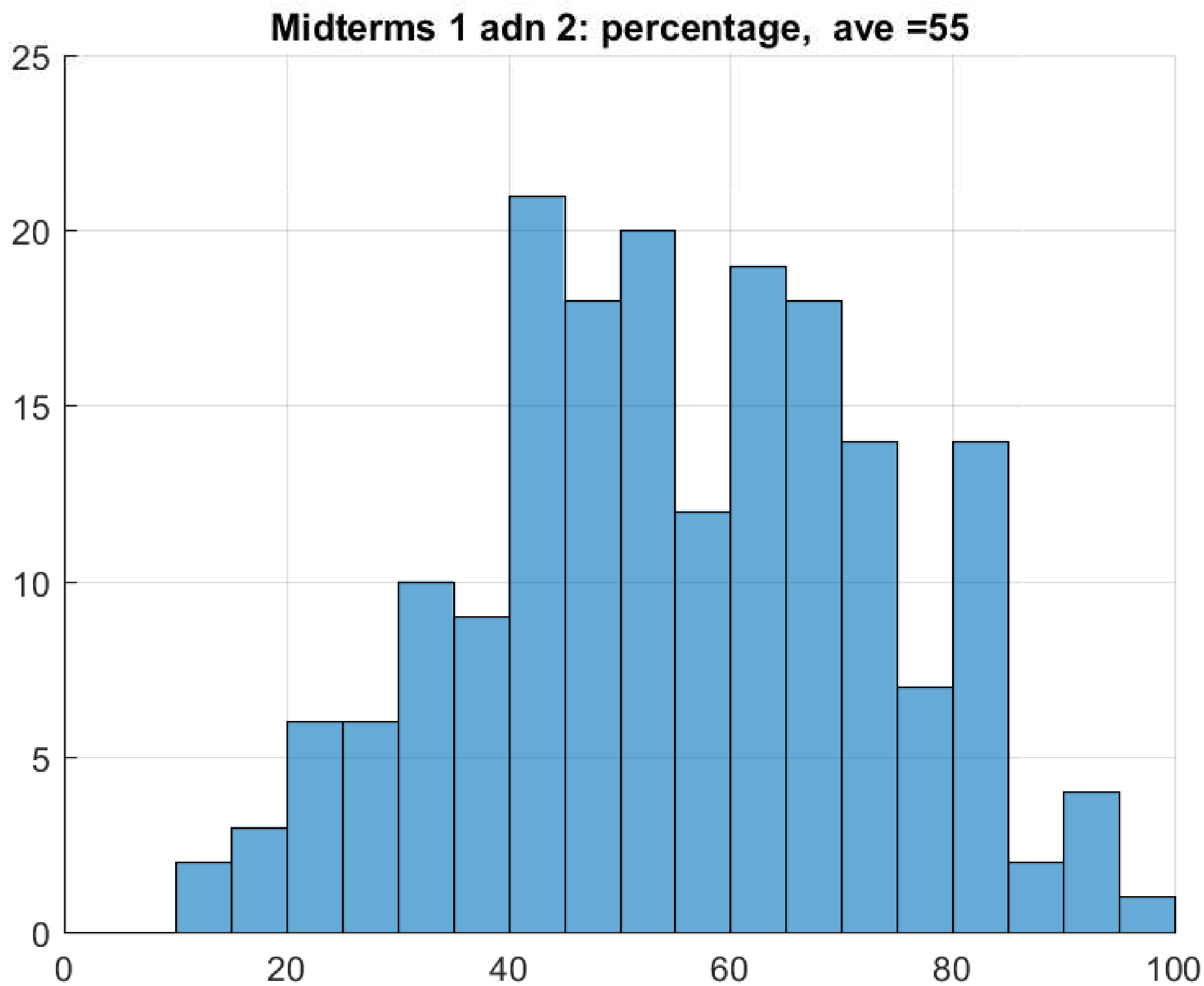
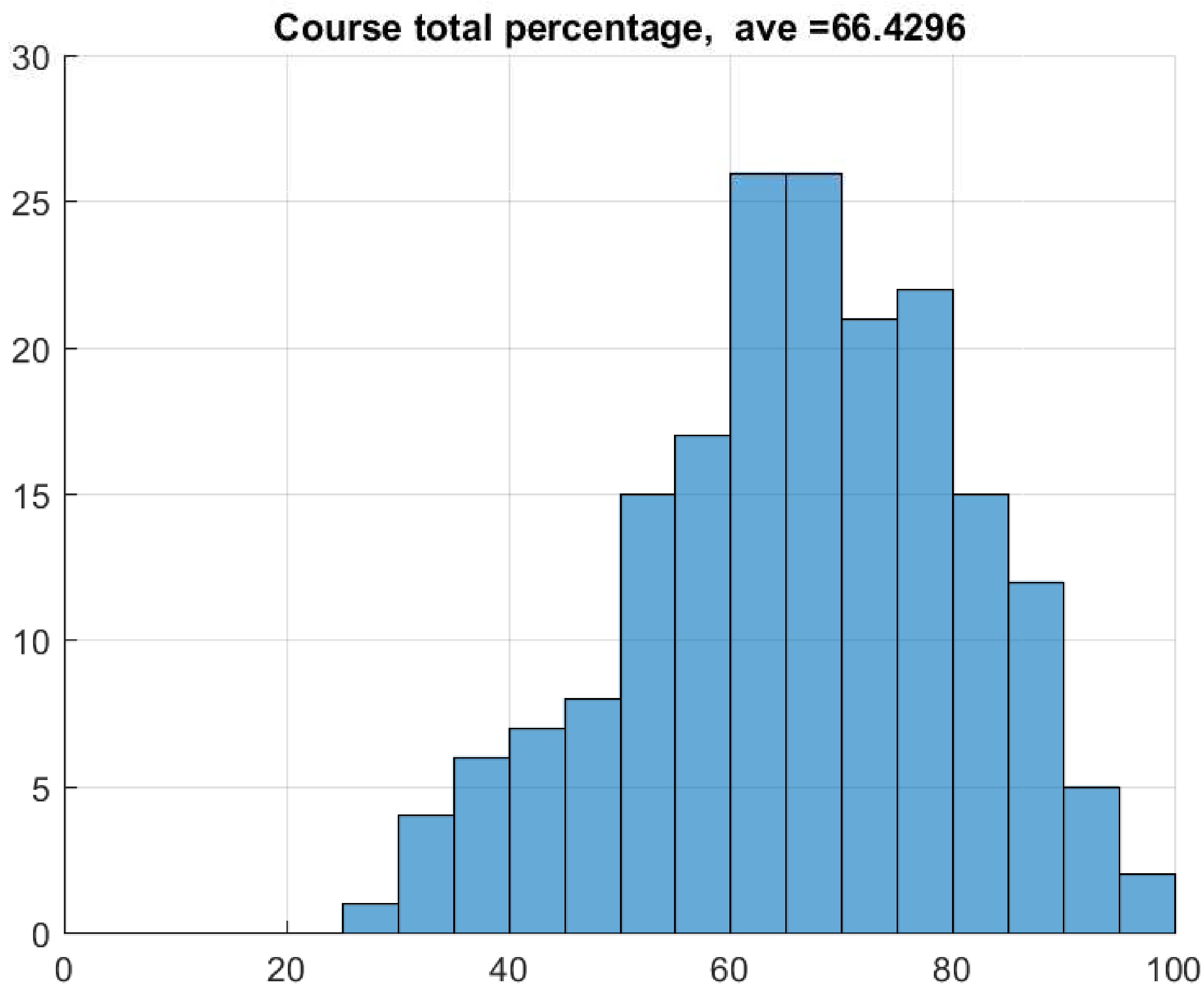

Quizzes 1-8, 2 dropped: percentage, ave =68.0669
30
25
20
15
10
5
0
0
20
40
60
80
100


Midterms 1 adn 2: percentage, ave =55
25
20
15
10
5
0
0
20
40
60
80
100



Course total percentage, ave =66.4296
30
25
20
15
10
5
0
0
20
40
60
80
100

Midterm is 25 points, 6 pages.

Several problems have multiple parts worth one point each.

No books or calculators are allowed.

One sheet (2-sided) of formulas and notes are allowed.

A sheet is on the webpage. You may bring it, or make your own.

- **Page 1:** 2 integration by parts, 1 theorem of Pappus
- **Page 2:** 3 problems on Newton's law of cooling.
- **Page 3:** 5 on center of mass. Compute area, integrals for M_y, M_x, give x, y coordinates.
- **Page 4:** 5 on partial fractions: long division, find A and B, integrate. Formula/Graph.
- **Page 5:** 4 problems: two indefinite integrals, then use to evaluate improper integral.
- **Page 6:** 5 on trig integrals: 3 choosing strategy. 2 setting up arclength, evaluate.

Page 1:

1. Integrate by parts: $\int xe^x dx$.

2. Use integration by parts to evaluate $\int x^2 \sin(2x) dx$.

3. Let S be a disk of radius 3 centered at $(x, y) = (4, 2)$. Use the theorem of Pappus to compute the volume obtained by rotating S around the y-axis.

Page 2:

4. A turkey at 60° is placed in a 400° degree oven. Give the formula for the turkey's temperature at time t according to Newton's law of cooling.

5. If the turkey in the previous problem is at 150° degrees after one hour, what is the value of $k > 0$?

6. Using the equation from Problem 4, when does the turkey reach 200°? (leave k as a symbol)

Page 3:

7. What is the area of the region $S = \{(x, y) : 0 \le x \le 1, x^3 \le y \le x\}$?

8. For the region above, what is the formula for M_y, the moment around the y-axis?

9. For the region above, what is the formula for M_x, the moment around the x-axis?

10. For the region above, what $\overline{x}$, the x-coordinate of the center of mass?

11. For the region above, what $\overline{y}$, the y-coordinate of the center of mass?

Page 4: Problems 12-15 all involve the same rational function and each step depends on the previous ones. Take extra care to check your answers, e.g., put your answers over a common denominator or plug in some values to check them.

12. Use long division of polynomials to write $r(x) = (2x^3+3x^2-4)/(x^2-4)$ in the form $p_1(x) + p_2(x)/q(x)$ where p_2 has lower degree than q.

Write the fractional part of r in its partial fraction expansion:

$$\frac{p_2(x)}{q(x)} = \frac{A}{X-2} + \frac{B}{X+2}.$$

13. Find A.

14. Find B.

15. Compute $\int_0^1 \frac{2x^3+3x^2-4}{x^2-4}dx$.

16. ☐ Which a possible partial fraction expansion for the graph below?

(a) $\frac{A}{x-4} + \frac{Bx+C}{(x+4)^2} + \frac{D}{1+(x+1)^2}$

(b) $\frac{A}{x+3} + \frac{Bx+C}{x^2} + \frac{D}{1+(x-3)^2}$

(c) $\frac{A}{x-2} + \frac{Bx+C}{(x+4)^2} + \frac{D}{1+(x+1)^2}$

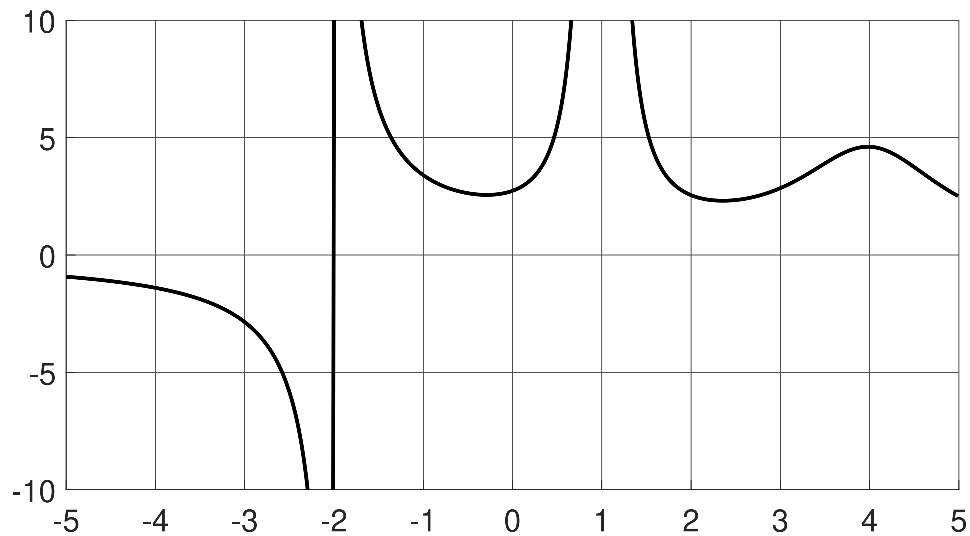
(d) $\frac{A}{x+2} + \frac{Bx+C}{(x-1)^2} + \frac{D}{1+(x-4)^2}$

(e) $\frac{A}{x+4} + \frac{Bx+C}{(x-4)^2} + \frac{D}{1+(x+1)^2}$

(f) $\frac{A}{x+4} + \frac{Bx+C}{(x-3)^2} + \frac{D}{1+x^2}$

(g) $\frac{A}{x+5} + \frac{Bx+C}{(x+2)^2} + \frac{D}{1+(x-4)^2}$

(h) none of these



Page 5:

17. Evaluate $\int x^3 e^{-x^2}\, dx$.

18. Evaluate the improper integral $\int_1^\infty x^3 e^{-x^2}\, dx$.

19. Evaluate $\int \frac{dx}{x\sqrt{\ln x}}$.

20. Evaluate the improper integral $\int_2^\infty \frac{dx}{x\sqrt{\ln x}}$.

Page 6: For each integral, select the appropriate strategy from the list.

21. ☐ $\int \cos^6 x \sin^9 x dx$ 22. ☐ $\int \tan^5 x dx$ 23. ☐ $\int \cos(10x)\cos(7x)dx$

Trigonometric integration strategies:

(a) Replace $\sin^2 x$ by $1 - \cos^2 x$ and the use substitution $u = \cos x$.

(b) Replace $\cos^2 x$ by $1 - \sin^2 x$ and the use substitution $u = \sin x$.

(c) Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, then (a) or (b).

(d) Use $\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$.

(e) Use $\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$.

(f) Use $\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$.

(g) Rewrite $\sec^j x = \sec^{j-2} x \sec^2 x$ and use $\sec^2 x = \tan^2 x + 1$. Then let $u = \tan x$.

(h) Rewrite $\tan^k x \sec^j x = \tan^{k-1} \sec^{j-1} x \tan x \sec x$ and use $\tan^2 = \sec^2 - 1$.
Then use $u = \sec x$

(i) Use $\tan^k x = \tan^{k-2} x \tan^2 x = \tan^{k-2} x(\sec^2 x - 1) = \tan^{k-2} x \sec^2 x - \tan^{k-2} x$.
Repeat if necessary.

(j) Use $\tan^2 x = \sec^2 x - 1$. Then integrate by parts the powers of $\sec x$.

24. What integral gives the arclength of $y = x^2$ over $[0, 1/2]$?

25. Use a trigonometric substitution to compute this arclength.