- **relative error:** for a tour with cost C, the ratio, $\frac{C - Opt}{Opt}$ (usually expressed in the form of a percentage), where Opt is the cost of an optimal tour, **188**
- **repetitive nearest-neighbor algorithm:** finds the nearest-neighbor tour for each possible starting vertex and chooses the one of least cost among them, **188, 190**
- **nearest-neighbor tour:** the tour obtained using the nearest-neighbor algorithm, **189**
- **repetitive nearest-neighbor tour:** a tour obtained using the nearest-neighbor algorithm, **190**

6.5 The Cheapest-Link Algorithm

- **partial-circuit rule:** a Hamilton circuit (tour) cannot contain any partial circuits, **191**
- **three-edge rule:** a Hamilton circuit (tour) cannot have three edges coming out of a vertex, **191**
- **cheapest-link algorithm:** at each step chooses the cheapest link available that does not violate the partial-circuit rule or the three-edge rule, **191**
- **cheapest-link tour:** a tour obtained using the cheapest-link algorithm, **191**

EXERCISES

WALKING

6.1 What Is a Traveling Salesman Problem?

No exercises for this section.

6.2 Hamilton Paths and Circuits

1. For the graph shown in Fig. 6-19,
   (a) find three different Hamilton circuits.
   (b) find a Hamilton path that starts at A and ends at B.
   (c) find a Hamilton path that starts at D and ends at F.

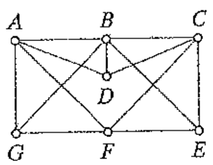


FIGURE 6-19

2. For the graph shown in Fig. 6-20,
   (a) find three different Hamilton circuits.
   (b) find a Hamilton path that starts at A and ends at B.
   (c) find a Hamilton path that starts at F and ends at I.

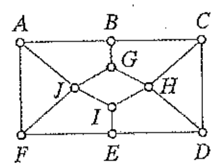


FIGURE 6-20

3. Find all possible Hamilton circuits in the graph in Fig. 6-21. Write your answers using A as the starting/ending vertex.

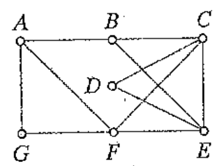


FIGURE 6-21

4. Find all possible Hamilton circuits in the graph in Fig. 6-22. Write your answers using A as the starting/ending vertex.

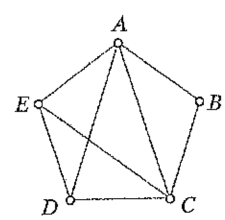


FIGURE 6-22

5. For the graph shown in Fig. 6-23,
   (a) find a Hamilton path that starts at *A* and ends at *E*.
   (b) find a Hamilton circuit that starts at *A* and ends with the edge *EA*.
   (c) find a Hamilton path that starts at *A* and ends at *C*.
   (d) find a Hamilton path that starts at *F* and ends at *G*.

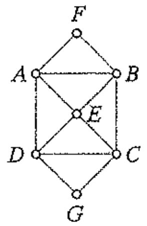


FIGURE 6-23

6. For the graph shown in Fig. 6-24,
   (a) find a Hamilton path that starts at *A* and ends at *E*.
   (b) find a Hamilton circuit that starts at *A* and ends with the edge *EA*.
   (c) find a Hamilton path that starts at *A* and ends at *G*.
   (d) find a Hamilton path that starts at *F* and ends at *G*.

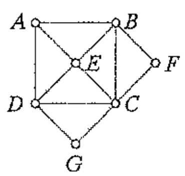


FIGURE 6-24

7. Suppose *D, G, E, A, H, C, B, F, D* is a Hamilton circuit in a graph.
   (a) Find the number of vertices in the graph.
   (b) Write the Hamilton circuit using *A* as the starting/ending vertex.
   (c) Find two different Hamilton paths in the graph that start at *A*.
8. Suppose *G, B, D, C, A, F, E, G* is a Hamilton circuit in a graph.
   (a) Find the number of vertices in the graph.
   (b) Write the Hamilton circuit using *F* as the starting/ending vertex.
   (c) Find two different Hamilton paths in the graph that start at *F*.
9. Consider the graph in Fig. 6-25.
   (a) Find the five Hamilton paths that can be obtained by "breaking" the Hamilton circuit *B,A,D,E,C,B* (i.e., by deleting just one edge from the circuit).
   (b) Find the eight Hamilton paths that do not come from "broken" Hamilton circuits (i.e., cannot be closed into a Hamilton circuit). (*Hint*: See Example 6.6).

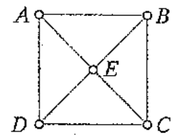


FIGURE 6-25

10. Consider the graph in Fig. 6-26.
    (a) Find all the Hamilton circuits in the graph, using *B* as the starting/ending vertex. (*Hint*: There are five Hamilton circuits and another five that are reversals of the first five.)
    (b) Find the four Hamilton paths that start at *B* and do not come from "broken" Hamilton circuits (i.e., cannot be closed into a Hamilton circuit).

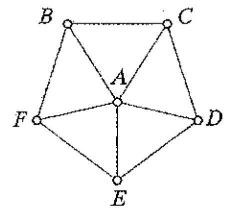


FIGURE 6-26

11. Consider the graph in Fig. 6-27.
    (a) Find all the Hamilton circuits in the graph, using *A* as the starting/ending vertex. You don't have to list both a circuit and its reversal—you can just list one from each pair.
    (b) Find all the Hamilton paths that do not come from "broken" Hamilton circuits (i.e., cannot be closed into a Hamilton circuit). You don't have to list both a path and its reversal—you can just list one from each pair.

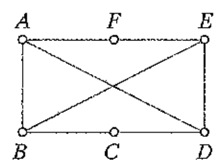


FIGURE 6-27

12. Consider the graph in Fig. 6-28.
    (a) Find all the Hamilton circuits in the graph, using *A* as the starting/ending vertex. You don't have to list both a circuit and its reversal—you can just list one from each pair.
    (b) Find all the Hamilton paths that do not come from "broken" Hamilton circuits (i.e., cannot be closed into a Hamilton circuit). You don't have to list both a path and its reversal—you can just list one from each pair. (*Hint*: Such paths must either start or end at *C*. You can just list all the paths that start at *C*— the ones that end at *C* are their reversals.)

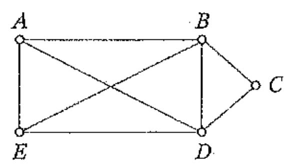


FIGURE 6-28

13. For the graph shown in Fig. 6-29,

(a) find a Hamilton path that starts at A and ends at F.

(b) find a Hamilton path that starts at K and ends at E.

(c) explain why the graph has no Hamilton path that starts at C.

(d) explain why the graph has no Hamilton circuits.

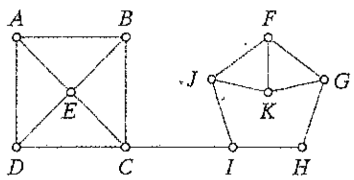


FIGURE 6-29

14. For the graph shown in Fig. 6-30,

(a) find a Hamilton path that starts at B.

(b) find a Hamilton path that starts at E.

(c) explain why the graph has no Hamilton path that starts at A or at C.

(d) explain why the graph has no Hamilton circuit.

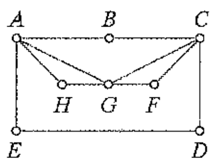


FIGURE 6-30

15. Explain why the graph shown in Fig. 6-31 has neither Hamilton circuits nor Hamilton paths.

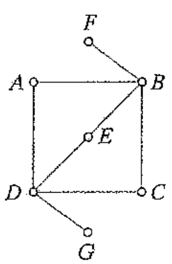


FIGURE 6-31

16. Explain why the graph shown in Fig. 6-32 has no Hamilton circuit but does have a Hamilton path.

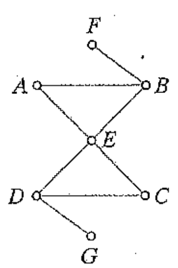


FIGURE 6-32

17. For the weighted graph shown in Fig. 6-33,

(a) find the weight of edge BD.

(b) find a Hamilton circuit that starts with edge BD, and give its weight.

(c) find a Hamilton circuit that ends with edge DB, and give its weight.

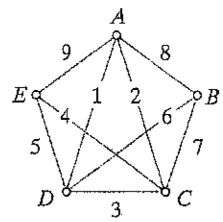


FIGURE 6-33

18. For the weighted graph shown in Fig. 6-34,

(a) find the weight of edge AD.

(b) find a Hamilton circuit that starts with edge AD, and give its weight.

(c) find a Hamilton circuit that ends with edge DA, and give its weight.

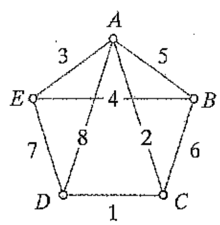


FIGURE 6-34

19. For the weighted graph shown in Fig. 6-35,

(a) find a Hamilton path that starts at A and ends at C, and give its weight.

(b) find a second Hamilton path that starts at A and ends at C, and give its weight.

(c) find the optimal (least weight) Hamilton path that starts at A and ends at C, and give its weight.

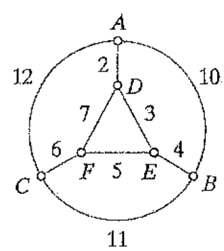


FIGURE 6-35

20. For the weighted graph shown in Fig. 6-36,

(a) find a Hamilton path that starts at B and ends at D, and give its weight.

(b) find a second Hamilton path that starts at B and ends at D, and give its weight.

(c) find the optimal (least weight) Hamilton path that starts at B and ends at D, and give its weight.

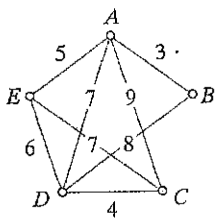


FIGURE 6-36

21. Suppose you have a supercomputer that can generate one *billion* Hamilton circuits per second.

(a) Estimate (in years) how long it would take the supercomputer to generate all the Hamilton circuits in K_{21}.

(b) Estimate (in years) how long it would take the supercomputer to generate all the Hamilton circuits in K_{22}.

22. Suppose you have a supercomputer that can generate one *trillion* Hamilton circuits per second.

(a) Estimate (in years) how long it would take the supercomputer to generate all the Hamilton circuits in K_{26}.

(b) Estimate (in years) how long it would take the supercomputer to generate all the Hamilton circuits in K_{27}.

23. (a) How many edges are there in K_{20}?

(b) How many edges are there in K_{21}?

(c) If the number of edges in K_{50} is x and the number of edges in K_{51} is y, what is the value of $y - x$?

24. (a) How many edges are there in K_{200}?

(b) How many edges are there in K_{201}?

(c) If the number of edges in K_{500} is x and the number of edges in K_{501} is y, what is the value of $y - x$?

25. In each case, find the value of N.

(a) K_N has 120 distinct Hamilton circuits.

(b) K_N has 45 edges.

(c) K_N has 20,100 edges.

26. In each case, find the value of N.

(a) K_N has 720 distinct Hamilton circuits.

(b) K_N has 66 edges.

(c) K_N has 80,200 edges.

6.3 The Brute-Force Algorithm

27. Find an optimal tour for the TSP given in Fig. 6-37, and give its cost.

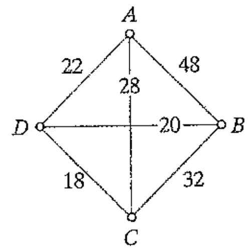


FIGURE 6-37

28. Find an optimal tour for the TSP given in Fig. 6-38, and give its cost.

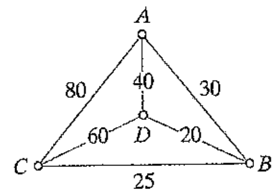


FIGURE 6-38

29. A truck must deliver furniture to stores located in five different cities A, B, C, D, and E. The truck must start and end its route at A. The time (in hours) for travel between the cities is given in Fig. 6-39. Find an optimal tour for this TSP and give its cost in hours. (*Hint*: The edge AD is part of an optimal tour.)

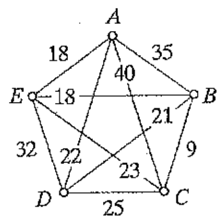


FIGURE 6-39

30. A social worker starts from her home A, must visit clients at B, C, D, and E (in any order), and return home to A at the end of the day. The graph in Fig. 6-40 shows the distance (in miles) between the five locations. Find an optimal tour for this TSP, and give its cost in miles. (*Hint*: The edge AC is part of an optimal tour.)

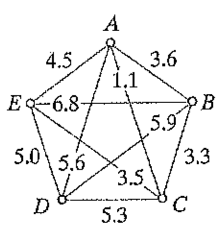


FIGURE 6-40

31. You are planning to visit four cities A, B, C, and D. Table 6-6 shows the time (in hours) that it takes to travel by car between any two cities. Find an optimal tour for this TSP that starts and ends at B.

	A	B	C	D
A	*	12	6	14
B	12	*	17	15
C	6	17	*	11
D	14	15	11	*

TABLE 6-6

32. An unmanned rover must be routed to visit four sites labeled A, B, C, and D on the surface of the moon. Table 6-7 shows the distance (in kilometers) between any two sites. Assuming the rover landed at C, find an optimal tour.

	A	B	C	D
A	0	4	18	16
B	4	0	17	13
C	18	17	0	7
D	16	13	7	0

TABLE 6-7

33. Consider a TSP with nine vertices labeled A through I.

(a) How many tours are of the form $A, G, \ldots, A$? (*Hint*: The remaining seven letters can be rearranged in any sequence.)

(b) How many tours are of the form $B, \ldots, E, B$?

(c) How many tours are of the form $A, D, \ldots, F, A$?

34. Consider a TSP with 11 vertices labeled A through K.

(a) How many tours are of the form $A, B, \ldots, A$? (*Hint*: The remaining nine letters can be rearranged in any sequence.)

(b) How many tours are of the form $C, \ldots, K, C$?

(c) How many tours are of the form $D, B, \ldots, K, D$?

6.4 The Nearest-Neighbor and Repetitive Nearest-Neighbor Algorithms

35. For the weighted graph shown in Fig. 6-41, (i) find the indicated tour, and (ii) give its cost. (*Note*: This is the TSP introduced in Example 6.1.)

(a) The nearest-neighbor tour with starting vertex B

(b) The nearest-neighbor tour with starting vertex C

(c) The nearest-neighbor tour with starting vertex D

(d) The nearest-neighbor tour with starting vertex E

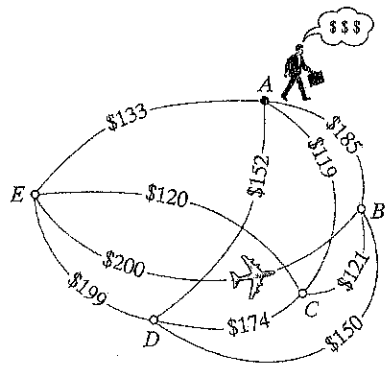


FIGURE 6-41

36. A delivery service must deliver packages at Buckman (B), Chatfield (C), Dayton (D), and Evansville (E) and then return to Arlington (A), the home base. Figure 6-42 shows a graph of the estimated travel times (in minutes) between the cities.

(a) Find the nearest-neighbor tour with starting vertex A. Give the total travel time of this tour.

(b) Find the nearest-neighbor tour with starting vertex D. Write the tour as it would be traveled if starting and ending at A. Give the total travel time of this tour.

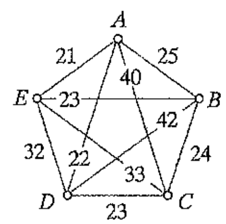


FIGURE 6-42

37. The Brute-Force Bandits is a rock band planning a five-city concert tour. The cities and the distances (in miles) between them are given in the weighted graph shown in Fig. 6-43. The tour must start and end at A. The cost of the chartered bus in which the band is traveling is $8 per mile.

(a) Find the nearest-neighbor tour with starting vertex A. Give the cost (in $) of this tour.

(b) Find the nearest-neighbor tour with starting vertex B. Write the tour as it would be traveled by the band, starting and ending at A. Give the cost (in $) of this tour.

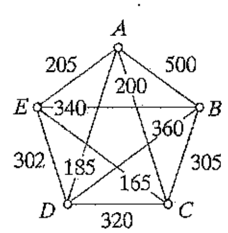


FIGURE 6-43

38. A space mission is scheduled to visit the moons Callisto (C), Ganymede (G), Io (I), Mimas (M), and Titan (T) to collect rock samples at each and then return to Earth (E). The travel times (in years) are given in the weighted graph shown in Fig. 6-44. (*Note*: This is the interplanetary TSP discussed in Example 6.11.)

(a) Find the nearest-neighbor tour with starting vertex E. Give the total travel time of this tour.

(b) Find the nearest-neighbor tour with starting vertex T. Write the tour as it would be traveled by an expedition starting and ending at E. Give the total travel time of this tour.

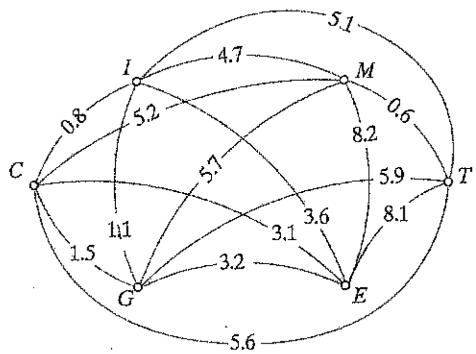


FIGURE 6-44

39. This exercise refers to the furniture truck TSP introduced in Exercise 29 (see Fig. 6-39).

(a) Find the nearest-neighbor tour starting at A.

(b) Find the nearest-neighbor tour starting at B, and give the answer using A as the starting/ending city.

40. This exercise refers to the social worker TSP introduced in Exercise 30 (see Fig. 6-40).

(a) Find the nearest-neighbor tour starting at A.

(b) Find the nearest-neighbor tour starting at C, and give the answer using A as the starting/ending city.

41. Darren is a sales rep whose territory consists of the six cities in the mileage chart shown in Fig. 6-45. Darren wants to visit customers at each of the cities, starting and ending his trip in his home city of Atlanta. His travel costs (gas, insurance, etc.) average $0.75 per mile.

(a) Find the nearest-neighbor tour with Atlanta as the starting city. What is the total cost of this tour?

(b) Find the nearest-neighbor tour using Kansas City as the starting city. Write the tour as it would be traveled by Darren, who must start and end the trip in Atlanta. What is the total cost of this tour?

Mileage Chart

	Atlanta	Columbus	Kansas City	Minneapolis	Pierre	Tulsa
Atlanta	*	533	798	1068	1361	772
Columbus	533	*	656	713	1071	802
Kansas City	798	656	*	447	592	248
Minneapolis	1068	713	447	*	394	695
Pierre	1361	1071	592	394	*	760
Tulsa	772	802	248	695	760	*

FIGURE 6-45

42. The Platonic Cowboys are a country and western band based in Nashville. The Cowboys are planning a concert tour to the seven cities in the mileage chart shown in Fig. 6-46.

(a) Find the nearest-neighbor tour with Nashville as the starting city. What is the total length of this tour?

(b) Find the nearest-neighbor tour using St. Louis as the starting city. Write the tour as it would be traveled by the band, which must start and end the tour in Nashville. What is the total length of this tour?

Mileage Chart

	Boston	Dallas	Houston	Louisville	Nashville	Pittsburgh	St. Louis
Boston	*	1748	1804	941	1088	561	1141
Dallas	1748	*	243	819	660	1204	630
Houston	1804	243	*	928	769	1313	779
Louisville	941	819	928	*	168	388	263
Nashville	1088	660	769	168	*	553	299
Pittsburgh	561	1204	1313	388	553	*	588
St. Louis	1141	630	779	263	299	588	*

FIGURE 6-46

43. Find the repetitive nearest-neighbor tour (and give its cost) for the furniture truck TSP discussed in Exercises 29 and 39 (see Fig. 6-39).

44. Find the repetitive nearest-neighbor tour for the social worker TSP discussed in Exercises 30 and 40 (see Fig. 6-40).

45. This exercise is a continuation of Darren's sales trip problem (Exercise 41). Find the repetitive nearest-neighbor tour, and give the total cost for this tour. Write the answer using Atlanta as the starting city.

46. This exercise is a continuation of the Platonic Cowboys concert tour (Exercise 42). Find the repetitive nearest-neighbor tour, and give the total mileage for this tour. Write the answer using Nashville as the starting city.

47. Suppose that in solving a TSP you use the nearest-neighbor algorithm and find a nearest-neighbor tour with a total cost of $13,500. Suppose that you later find out that the cost of an optimal tour is $12,000. What was the relative error of your nearest-neighbor tour? Express your answer as a percentage, rounded to the nearest tenth of a percent.

48. Suppose that in solving a TSP you use the nearest-neighbor algorithm and find a nearest-neighbor tour with a total length of 21,400 miles. Suppose that you later find out that the length of an optimal tour is 20,100 miles. What was the relative error of your nearest-neighbor tour? Express your answer as a percentage, rounded to the nearest tenth of a percent.

6.5 Cheapest-Link Algorithm

49. Find the cheapest-link tour (and give its cost) for the furniture truck TSP discussed in Exercise 29 (see Fig. 6-39).

50. Find the cheapest-link tour for the social worker TSP discussed in Exercise 30 (see Fig. 6-40).

51. For the Brute-Force Bandits concert tour discussed in Exercise 37, find the cheapest-link tour, and give the bus cost for this tour (see Fig. 6-43).

52. For the weighted graph shown in Fig. 6-47, find the cheapest-link tour. Write the tour using *B* as the starting vertex.

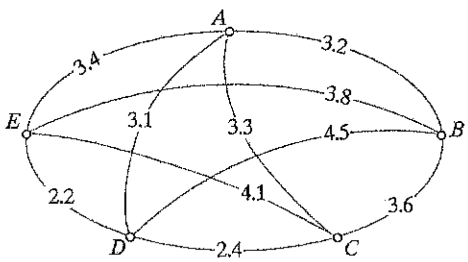


FIGURE 6-47

53. For Darren's sales trip problem discussed in Exercise 41, find the cheapest-link tour, and give the total cost for this tour (see Fig. 6-45).

54. For the Platonic Cowboys concert tour discussed in Exercise 42, find the cheapest-link tour, and give the total mileage for this tour (see Fig. 6-46).

55. A rover on the planet Mercuria has to visit six sites labeled *A* through *F*. Figure 6-48 shows the time (in days) for the rover to travel between any two sites.

(a) Find the cheapest-link tour for these sites and give its length.

(b) Given that the tour *A, B, D, F, C, E, A* is an optimal tour, find the relative error of the cheapest-link tour found in (a).

	A	*B*	*C*	*D*	*E*	*F*
A	*	11	19	16	9	10
B	11	*	20	13	17	15
C	19	20	*	21	13	11
D	16	13	21	*	12	16
E	9	17	13	12	*	14
F	10	15	11	16	14	*

FIGURE 6-48

56. A robotic laser must drill holes on five sites (*A, B, C, D*, and *E*) in a microprocessor chip. At the end, the laser must return to its starting position *A* and start all over. Figure 6-49 shows the time (in seconds) it takes the laser arm to move from one site to another. In this TSP, a tour is a sequence of drilling locations starting and ending at *A*.

(a) Find the cheapest-link tour and its length.

(b) Given that the tour *A, D, B, E, C, A* is an optimal tour, find the relative error of the cheapest-link tour found in (a).

	A	*B*	*C*	*D*	*E*
A	*	1.2	0.7	1.0	1.3
B	1.2	*	0.9	0.8	1.1
C	0.7	0.9	*	1.2	0.8
D	1.0	0.8	1.2	*	0.9
E	1.3	1.1	0.8	0.9	*

FIGURE 6-49

JOGGING

57. Suppose that in solving a TSP you find an approximate solution with a cost of $1614, and suppose that you later find out that the relative error of your solution was 7.6%. What was the cost of the optimal solution?

58. Suppose that in solving a TSP you find an approximate solution with a cost of $2508, and suppose that you later find out that the relative error of your solution was 4.5%. What was the cost of the optimal solution?

59. You have a busy day ahead of you. You must run the following errands (in no particular order): Go to the post office, deposit a check at the bank, pick up some French bread at the deli, visit a friend at the hospital, and get a haircut at Karl's Beauty Salon. You must start and end at home. Each block on the map shown in Fig. 6-50 is exactly 1 mile.

(a) Draw a weighted graph modeling to this problem.

(b) Find an optimal tour for running all the errands. (Use any algorithm you think is appropriate.)

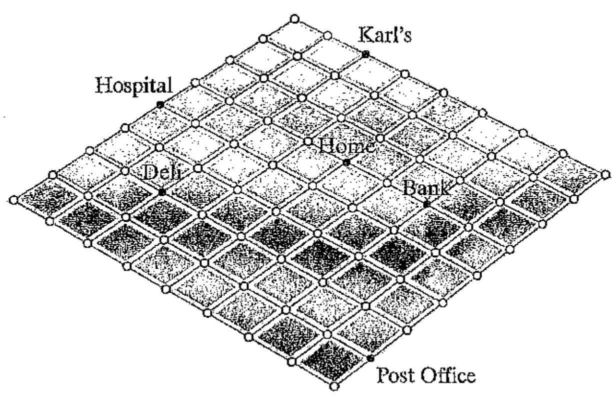


FIGURE 6-50

In Exercises 60 and 61, you are scheduling a dinner party for six people (A, B, C, D, E, and F). The guests are to be seated around a circular table, and you want to arrange the seating so that each guest is seated between two friends (i.e., the guests to the left and to the right are friends of the guest in between). You can assume that all friendships are mutual (when X is a friend of Y, Y is also a friend of X).

60. Suppose that you are told that all possible friendships can be deduced from the following information:

A is friends with B and F; B is friends with A, C, and E; C is friends with B, D, E, and F; E is friends with B, C, D, and F.

(a) Draw a "friendship graph" for the dinner guests.

(b) Find a possible seating arrangement for the party.

(c) Is there a possible seating arrangement in which B and E are seated next to each other? If there is, find it. If there isn't, explain why not.

61. Suppose that you are told that all possible friendships can be deduced from the following information:

A is friends with C, D, E, and F; B is friends with C, D, and E; C is friends with A, B, and E; D is friends with A, B, and E.

Explain why it is impossible to have a seating arrangement in which each guest is seated between friends.

62. If the number of edges in K_{500} is x and the number of edges in K_{502} is y, what is the value of $y - x$?

63. A 2 by 2 grid graph. The graph shown in Fig. 6-51 represents a street grid that is 2 blocks by 2 blocks. (Such a graph is called a *2 by 2 grid graph.*) For convenience, the vertices are labeled by type: corner vertices C_1, C_2, C_3, and C_4, boundary vertices B_1, B_2, B_3, and B_4, and the interior vertex I.

(a) Find a Hamilton path in the graph that starts at I.

(b) Find a Hamilton path in the graph that starts at one of the corner vertices and ends at a different corner vertex.

(c) Find a Hamilton path that starts at one of the corner vertices and ends at I.

(d) Find (if you can) a Hamilton path that starts at one of the corner vertices and ends at one of the boundary vertices. If this is impossible, explain why.

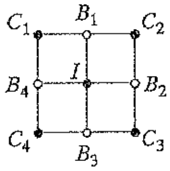


FIGURE 6-51

64. Find (if you can) a Hamilton circuit in the 2 by 2 grid graph discussed in Exercise 63. If this is impossible, explain why.

65. A 3 by 3 grid graph. The graph shown in Fig. 6-52 represents a street grid that is 3 blocks by 3 blocks. The graph has four corner vertices (C_1, C_2, C_3, and C_4), eight boundary vertices (B_1 through B_8), and four interior vertices (I_1, I_2, I_3, and I_4).

(a) Find a Hamilton circuit in the graph.

(b) Find a Hamilton path in the graph that starts at one of the corner vertices and ends at a different corner vertex.

(c) Find (if you can) a Hamilton path that starts at one of the corner vertices and ends at one of the interior vertices. If this is impossible, explain why.

(d) Given any two adjacent vertices of the graph, explain why there always is a Hamilton path that starts at one and ends at the other one.

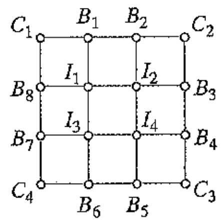


FIGURE 6-52

66. A 3 by 4 grid graph. The graph shown in Fig. 6-53 represents a street grid that is 3 blocks by 4 blocks.

(a) Draw a Hamilton circuit in the graph.

(b) Draw a Hamilton path in the graph that starts at C_1 and ends at C_3.

(c) Draw (if you can) a Hamilton path in the graph that starts at C_1 and ends at C_2. If this is impossible, explain why.

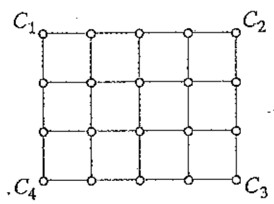


FIGURE 6-53

67. Explain why the cheapest edge in any graph is always part of the Hamilton circuit obtained using the nearest-neighbor algorithm.

68. (a) Explain why a graph that has a bridge cannot have a Hamilton circuit.

(b) Give an example of a graph with bridges that has a Hamilton path.

69. Nick is a traveling salesman. His territory consists of the 11 cities shown on the mileage chart in Fig. 6-54. Nick must find a tour that starts and ends in Dallas (that's his home) and visits each of the other 10 cities.

(a) Find a nearest-neighbor tour that starts at Dallas.

(b) Find the cheapest-link tour.

Mileage Chart

	Atlanta	Boston	Buffalo	Chicago	Columbus	Dallas	Denver	Houston	Kansas City	Louisville	Memphis
Atlanta	*	1037	859	674	533	795	1398	789	798	382	371
Boston	1037	*	446	963	735	1748	1949	1804	1391	941	1293
Buffalo	859	446	*	522	326	1346	1508	1460	966	532	899
Chicago	674	963	522	*	308	917	996	1067	499	292	530
Columbus	533	735	326	308	*	1028	1229	1137	656	209	576
Dallas	795	1748	1346	917	1028	*	781	243	489	819	452
Denver	1398	1949	1508	996	1229	781	*	1019	600	1120	1040
Houston	789	1804	1460	1067	1137	243	1019	*	710	928	561
Kansas City	798	1391	966	499	656	489	600	710	*	520	451
Louisville	382	941	532	292	209	819	1120	928	520	*	367
Memphis	371	1293	899	530	576	452	1040	561	451	367	*

FIGURE 6-54

70. Julie is the marketing manager for a small software company based in Boston. She is planning a sales trip to Michigan to visit customers in each of the nine cities shown on the mileage chart in Fig. 6-55. She can fly from Boston to any one of the cities and fly out of any one of the cities back to Boston for the same price (call the arrival city A and the departure city D). Her plan is to pick up a rental car at A, drive to each of the other cities, and drop off the rental car at the last city D. Slightly complicating the situation is that Michigan has two separate peninsulas—an upper peninsula and a lower peninsula—and the only way to get from one to the other is through the Mackinaw Bridge connecting Cheboygan to Sault Ste. Marie. (There is a $3 toll to cross the bridge in either direction.)

Mileage Chart

	Detroit	Lansing	Grand Rapids	Flint	Cheboygan	Sault Ste. Marie	Marquette	Escanaba	Menominee
Detroit	*	90	158	68	280				
Lansing	90	*	68	56	221				
Grand Rapids	158	68	*	114	233				
Flint	68	56	114	*	215				
Cheboygan	280	221	233	215	*	78			
Sault Ste. Marie					78	*	164	174	227
Marquette						164	*	67	120
Escanaba						174	67	*	55
Menominee						227	120	55	*

FIGURE 6-55 .

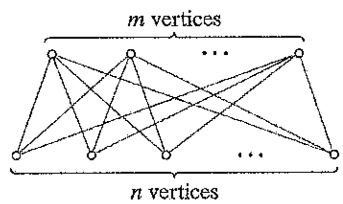
(a) Suppose that the rental car company charges 39 cents per mile plus a drop off fee of $250 if A and D are different cities (there is no charge if $A = D$). Find the optimal (cheapest) route and give the total cost.

(b) Suppose that the rental car company charges 49 cents per mile but the car can be returned to any city without a drop off fee. Find the optimal route and give the total cost.

RUNNING

71. Complete bipartite graphs. A complete bipartite graph is a graph with the property that the vertices can be divided into two sets A and B and each vertex in set A is adjacent to each of the vertices in set B. There are no other edges! If there are m vertices in set A and n vertices in set B, the complete bipartite graph is written as $K_{n,n}$. Figure 6-56 shows a generic bipartite graph.

(a) For $n > 1$, the complete bipartite graphs of the form $K_{m,n}$ all have Hamilton circuits. Explain why.



$K_{m,n}$

FIGURE 6-56

(b) If the difference between m and n is exactly 1 (i.e., $|m - n| = 1$), the complete bipartite graph $K_{m,n}$ has a Hamilton path. Explain why.

(c) When the difference between m and n is more than 1, then the complete bipartite graph $K_{m,n}$ has neither a Hamilton circuit nor a Hamilton path. Explain why.

72. *m* by *n* grid graphs. An m by n grid graph represents a rectangular street grid that is m blocks by n blocks, as indicated in Fig. 6-57. (You should try Exercises 63 through 66 before you try this one.)

(a) If m and n are both odd, then the m by n grid graph has a Hamilton circuit. Describe the circuit by drawing it on a generic graph.

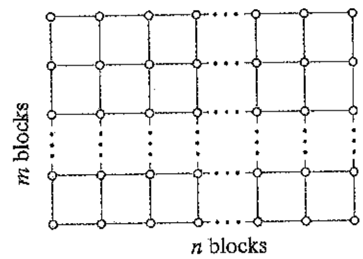


FIGURE 6-57

(b) If either m or n is even and the other one is odd, then the m by n grid graph has a Hamilton circuit. Describe the circuit by drawing it on a generic graph.

(c) If m and n are both even, then the m by n grid graph does not have a Hamilton circuit. Explain why a Hamilton circuit is impossible.

73. Ore's theorem. A connected graph with N vertices is said to satisfy *Ore's condition* if $\deg(X) + \deg(Y) \geq N$ for every pair of vertices X and Y of the graph. Ore's theorem states that *if a graph satisfies Ore's condition, then it has a Hamilton circuit.*

(a) Explain why the complete bipartite graph $K_{n,n}$ (see Exercise 71) satisfies Ore's condition.

(b) Explain why for $m \neq n$, the complete bipartite graph $K_{m,n}$ (see Exercise 71) does *not* satisfy Ore's condition.

(c) Ore's condition is sufficient to guarantee that a connected graph has a Hamilton circuit but is not a necessary condition. Give an example of a graph that has a Hamilton circuit but does not satisfy Ore's condition.

74. Dirac's theorem. If G is a connected graph with N vertices and $\deg(X) \geq \frac{N}{2}$ for every vertex X, then *G has a Hamilton circuit.* Explain why Dirac's theorem is a direct consequence of Ore's theorem.

PROJECTS AND PAPERS

1 The Nearest-Insertion Algorithm

The *nearest-insertion algorithm* is another approximate algorithm used for tackling TSPs. The basic idea of the algorithm is to start with a subcircuit (a circuit that includes some, but not all, of the vertices) and enlarge it, one step at a time, by adding an extra vertex—the one that is closest to some vertex in the circuit. By the time we have added all of the vertices, we have a full-fledged Hamilton circuit.

In this project, you should prepare a class presentation on the nearest-insertion algorithm. Your presentation should include a detailed description of the algorithm, at least two carefully worked-out examples, and a comparison of the nearest-insertion and the nearest-neighbor algorithms.

2 Computing with DNA

DNA is the basic molecule of life—it encodes the genetic information that characterizes all living organisms. Due to the recent great advances in biochemistry, scientists can now snip, splice, and recombine segments of DNA almost at will. In 1994, Leonard Adleman, a professor of computer science at the University of Southern California, was able to encode a graph representing seven cities into a set of DNA segments and to use the chemical reactions of the DNA fragments to uncover the existence of a Hamilton path in the graph. Basically, he was able to use the biochemistry of DNA to solve a graph theory problem. While the actual problem solved was insignificant, the idea was revolutionary, as it opened the door for the possibility of someday using DNA computers to solve problems beyond the reach of even the most powerful of today's electronic computers.

Write a research paper telling the story of Adleman's landmark discovery. How did he encode the graph into DNA? How did he extract the mathematical solution (Hamilton path) from the chemical solution? What other kinds of problems might be solved using DNA computing? What are the implications of Adleman's discovery for the future of computing?