EXERCISES

WALKING

1.1 Ballots and Preference Schedules

1. Figure 1-8 shows the preference ballots for an election with 21 voters and 5 candidates. Write out the preference schedule for this election.

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *C*	1st *A*	1st *B*	1st *A*	1st *C*	1st *D*	1st *A*
2nd *E*	2nd *D*	2nd *E*	2nd *B*	2nd *E*	2nd *C*	2nd *B*
3rd *D*	3rd *B*	3rd *A*	3rd *C*	3rd *D*	3rd *B*	3rd *C*
4th *A*	4th *C*	4th *C*	4th *D*	4th *A*	4th *E*	4th *D*
5th *B*	5th *E*	5th *D*	5th *E*	5th *B*	5th *A*	5th *E*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *B*	1st *A*	1st *D*	1st *D*	1st *A*	1st *C*	1st *A*
2nd *E*	2nd *B*	2nd *C*	2nd *C*	2nd *B*	2nd *E*	2nd *D*
3rd *A*	3rd *C*	3rd *B*	3rd *B*	3rd *C*	3rd *D*	3rd *B*
4th *C*	4th *D*	4th *A*	4th *E*	4th *D*	4th *A*	4th *C*
5th *D*	5th *E*	5th *E*	5th *A*	5th *E*	5th *B*	5th *E*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *B*	1st *C*	1st *A*	1st *C*	1st *A*	1st *D*	1st *D*
2nd *E*	2nd *E*	2nd *B*	2nd *E*	2nd *D*	2nd *C*	2nd *C*
3rd *A*	3rd *D*	3rd *C*	3rd *D*	3rd *B*	3rd *B*	3rd *B*
4th *C*	4th *A*	4th *D*	4th *A*	4th *C*	4th *A*	4th *E*
5th *D*	5th *B*	5th *E*	5th *B*	5th *E*	5th *E*	5th *A*

FIGURE 1-8

2. Figure 1-9 shows the preference ballots for an election with 17 voters and 4 candidates. Write out the preference schedule for this election.

Ballot	Ballot	Ballot	Ballot	Ballot
1st *C*	1st *B*	1st *A*	1st *C*	1st *B*
2nd *A*	2nd *C*	2nd *D*	2nd *A*	2nd *C*
3rd *D*	3rd *D*	3rd *B*	3rd *D*	3rd *D*
4th *B*	4th *A*	4th *C*	4th *B*	4th *A*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *A*	1st *A*	1st *B*	1st *B*	1st *C*	1st *C*
2nd *D*	2nd *C*	2nd *C*	2nd *C*	2nd *A*	2nd *A*
3rd *B*	3rd *D*	3rd *D*	3rd *D*	3rd *D*	3rd *D*
4th *C*	4th *B*	4th *A*	4th *A*	4th *B*	4th *B*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *A*	1st *A*	1st *C*	1st *B*	1st *A*	1st *C*
2nd *C*	2nd *D*	2nd *A*	2nd *C*	2nd *D*	2nd *A*
3rd *D*	3rd *B*	3rd *D*	3rd *D*	3rd *B*	3rd *D*
4th *B*	4th *C*	4th *B*	4th *A*	4th *C*	4th *B*

FIGURE 1-9

Exercises 3 through 6 refer to an alternative format for preference ballots in which the names of the candidates appear on the ballot and the voter is asked to put a rank (1, 2, 3, etc.) next to each name [see Fig. 1-1(c)]. (This alternative format makes it easier on the voters and is useful when the names are long or when a misspelled name invalidates the ballot. The main disadvantage is that it tends to favor the candidates that are listed first.)

3. Table 1-25 shows the preference schedule for an election based on the alternative format for the preference ballots. Rewrite Table 1-25 in the conventional preference schedule format used in the text. (Use *A*, *B*, *C*, *D*, and *E* as shorthand for the names of the candidates.)

Number of voters	37	36	24	13	5
Alvarez	3	1	2	4	3
Brownstein	1	2	1	2	5
Clarkson	4	4	5	3	1
Dax	5	3	3	5	4
Easton	2	5	4	1	2

TABLE 1-25

4. Table 1-26 shows the preference schedule for an election based on the alternative format for the preference ballots. Rewrite Table 1-26 in the conventional preference schedule format used in the text. (Use *A*, *B*, *C*, *D*, and *E* as shorthand for the names of the candidates.)

Number of voters	14	10	8	7	4
Andersson	2	3	1	5	3
Broderick	1	1	2	3	2
Clapton	4	5	5	2	4
Dutkiewicz	5	2	4	1	5
Eklundh	3	4	3	4	1

TABLE 1-26

5. Table 1-27 shows the preference schedule for an election. Rewrite Table 1-27 using the alternative preference schedule format.

Number of voters	14	10	8	7	4
1st	*B*	*B*	*A*	*D*	*E*
2nd	*A*	*D*	*B*	*C*	*B*
3rd	*E*	*A*	*E*	*B*	*A*
4th	*D*	*E*	*D*	*E*	*C*
5th	*C*	*C*	*C*	*A*	*D*

TABLE 1-27

6. Table 1-28 shows the preference schedule for an election. Rewrite Table 1-28 using the alternative preference schedule format.

Number of voters	37	36	24	13	5
1st	*A*	*B*	*D*	*C*	*B*
2nd	*C*	*A*	*B*	*A*	*D*
3rd	*B*	*D*	*C*	*E*	*E*
4th	*E*	*C*	*E*	*B*	*A*
5th	*D*	*E*	*A*	*D*	*C*

■ TABLE 1-28

7. An election is held to choose the Chair of the Mathematics Department at Tasmania State University. The candidates are Professors Argand, Brandt, Chavez, Dietz, and Epstein (*A*, *B*, *C*, *D*, and *E* for short). Table 1-29 shows the preference schedule for the election.

Number of voters	5	5	3	3	3	2
1st	*A*	*C*	*A*	*B*	*D*	*D*
2nd	*B*	*E*	*D*	*E*	*C*	*C*
3rd	*C*	*D*	*B*	*A*	*B*	*B*
4th	*D*	*A*	*C*	*C*	*E*	*A*
5th	*E*	*B*	*E*	*D*	*A*	*E*

■ TABLE 1-29

(a) How many people voted in this election?

(b) How many first-place votes are needed for a majority?

(c) Which candidate had the fewest last-place votes?

8. The student body at Eureka High School is having an election for Homecoming Queen. The candidates are Alicia, Brandy, Cleo, and Dionne (*A*, *B*, *C*, and *D* for short). Table 1-30 shows the preference schedule for the election.

Number of voters	202	160	153	145	125	110	108	102	55
1st	*B*	*C*	*A*	*D*	*D*	*C*	*B*	*A*	*A*
2nd	*D*	*B*	*C*	*B*	*A*	*A*	*C*	*B*	*D*
3rd	*A*	*A*	*B*	*A*	*C*	*D*	*A*	*D*	*C*
4th	*C*	*D*	*D*	*C*	*B*	*B*	*D*	*C*	*B*

■ TABLE 1-30

(a) How many students voted in this election?

(b) How many first-place votes are needed for a majority?

(c) Which candidate had the fewest last-place votes?

9. The Demublican Party is holding its annual convention. The 1500 voting delegates are choosing among three possible party platforms: *L* (a liberal platform), *C* (a conservative platform), and *M* (a moderate platform). Seventeen percent of the delegates prefer *L* to *M* and *M* to *C*. Thirty-two percent of the delegates like *C* the most and *L* the least. The rest of the delegates like *M* the most and *C* the least. Write out the preference schedule for this election.

10. The Epicurean Society is holding its annual election for president. The three candidates are *A*, *B*, and *C*. Twenty percent of the voters like *A* the most and *B* the least. Forty percent of the voters like *B* the most and *A* the least. Of the remaining voters 225 prefer *C* to *B* and *B* to *A*, and 675 prefer *C* to *A* and *A* to *B*. Write out the preference schedule for this election.

1.2 Plurality Method

11. Table 1-31 shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

Number of voters	27	15	11	9	8	1
1st	*C*	*A*	*B*	*D*	*B*	*B*
2nd	*D*	*B*	*D*	*A*	*A*	*A*
3rd	*B*	*D*	*A*	*B*	*C*	*D*
4th	*A*	*C*	*C*	*C*	*D*	*C*

■ TABLE 1-31

12. Table 1-32 shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

Number of voters	29	21	18	10	1
1st	*D*	*A*	*B*	*C*	*C*
2nd	*C*	*C*	*A*	*B*	*B*
3rd	*A*	*B*	*C*	*A*	*D*
4th	*B*	*D*	*D*	*D*	*A*

■ TABLE 1-32

13. Table 1-33 shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

Number of voters	6	5	4	2	2	2	2
1st	*C*	*A*	*B*	*B*	*C*	*C*	*C*
2nd	*D*	*D*	*D*	*A*	*B*	*B*	*D*
3rd	*A*	*C*	*C*	*C*	*A*	*D*	*B*
4th	*B*	*B*	*A*	*D*	*D*	*A*	*A*

TABLE 1-33

14. Table 1-34 shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

Number of voters	6	6	5	4	3	3
1st	*A*	*B*	*B*	*D*	*A*	*B*
2nd	*C*	*C*	*C*	*A*	*C*	*A*
3rd	*D*	*A*	*D*	*C*	*D*	*C*
4th	*B*	*D*	*A*	*B*	*B*	*D*

TABLE 1-34

15. Table 1-35 shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

Percent of voters	24	23	19	14	11	9
1st	*C*	*D*	*D*	*B*	*A*	*D*
2nd	*A*	*A*	*A*	*C*	*C*	*C*
3rd	*B*	*C*	*E*	*A*	*B*	*A*
4th	*E*	*B*	*C*	*D*	*E*	*E*
5th	*D*	*E*	*B*	*E*	*D*	*B*

TABLE 1-35

16. Table 1-36 shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

Percent of voters	25	21	15	12	10	9	8
1st	*C*	*E*	*B*	*A*	*C*	*C*	*C*
2nd	*E*	*D*	*D*	*D*	*D*	*B*	*E*
3rd	*D*	*B*	*E*	*B*	*E*	*A*	*D*
4th	*A*	*A*	*C*	*E*	*A*	*E*	*B*
5th	*B*	*C*	*A*	*C*	*B*	*D*	*A*

TABLE 1-36

17. Table 1-29 (see Exercise 7) shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). In this election ties are not allowed to stand, and the following tie-breaking rule is used: *Whenever there is a tie between candidates, the tie is broken in favor of the candidate with the fewer last-place votes.* Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

18. Table 1-30 (see Exercise 8) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). In this election ties are not allowed to stand, and the following tie-breaking rule is used: *Whenever there is a tie between candidates, the tie is broken in favor of the candidate with the fewer last-place votes.* Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

19. Table 1-29 (see Exercise 7) shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). In this election ties are not allowed to stand, and the following tie-breaking rule is used: *Whenever there is a tie between two candidates, the tie is broken in favor of the winner of a head-to-head comparison between the candidates.* Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

20. Table 1-30 (see Exercise 8) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). In this election ties are not allowed to stand, and the following tie-breaking rule is used: *Whenever there is a tie between two candidates, the tie is broken in favor of the winner of a head-to-head comparison between the candidates.* Use the plurality method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

1.3 Borda Count

21. Table 1-31 (see Exercise 11) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the Borda count method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

22. Table 1-32 (see Exercise 12) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the Borda count method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

23. Table 1-33 (see Exercise 13) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the Borda count method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

24. Table 1-34 (see Exercise 14) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the Borda count method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

25. Table 1-35 (see Exercise 15) shows the preference schedule for an election with five candidates (A, B, C, D, and E). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Use the Borda count method to find the complete ranking of the candidates. (*Hint*: The ranking does not depend on the number of voters, so you can pick any convenient number to use for the number of voters.)

26. Table 1-36 (see Exercise 16) shows the preference schedule for an election with five candidates (A, B, C, D, and E). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Use the Borda count method to find the complete ranking of the candidates. (*Hint*: The ranking does not depend on the number of voters, so you can pick any convenient number to use for the number of voters.)

27. The 2009 Heisman Award. Table 1-37 shows the results of the balloting for the 2009 Heisman Award. Find the ranking of the top four finalists and the number of points each one received (see Example 1.11).

Player	School	1st	2nd	3rd
Toby Gerhart	Stanford	222	225	160
Mark Ingram	Alabama	227	236	151
Colt McCoy	Texas	203	188	160
Ndamukong Suh	Nebraska	161	105	122

Source: Heisman Award, *www.heisman.com/winners/m-ingram09.php*

■ TABLE 1-37

28. The 2011 NL Cy Young Award. Table 1-38 shows the top 5 finalists for the 2011 National League Cy Young Award. Find the ranking of the top 5 finalists and the number of points each one received (see Example 1.12).

Pitcher	1st	2nd	3rd	4th	5th
Roy Halladay (PHI)	4	21	7	0	0
Cole Hamels (PHI)	0	0	0	2	13
Ian Kennedy (AZ)	1	3	6	18	3
Clayton Kershaw (LA)	27	3	2	0	0
Cliff Lee (PHI)	0	5	17	9	1

Source: Baseball-Reference.com, *www.baseball-reference.com/awards/awards_2011.shtml*

■ TABLE 1-38

29. An election was held using the Borda count method. There were four candidates (A, B, C, and D) and 110 voters. When the points were tallied (using 4 points for first, 3 points for second, 2 points for third, and 1 point for fourth), A had 320 points, B had 290 points, and C had 180 points. Find how many points D had and give the ranking of the candidates. (*Hint*: Figure out how many points are packed in each ballot.)

30. An election was held using the following variation of the Borda count method: 7 points for first-place, 4 points for second, 3 points for third, 2 points for fourth, and 1 point for fifth. There were five candidates (A, B, C, D, and E) and 30 voters. When the points were tallied A had 84 points, B had 65 points, C had 123 points, and D had 107 points. Find how many points E had and give the ranking of the candidates. (*Hint*: Figure out how many points are packed in each ballot.)

1.4 Plurality-with-Elimination

31. Table 1-31 (see Exercise 11) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality-with-elimination method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

32. Table 1-32 (see Exercise 12) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality-with-elimination method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

33. Table 1-33 (see Exercise 13) shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality-with-elimination method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

34. Table 1-34 (see Exercise 14) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the plurality-with-elimination method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

35. Table 1-35 (see Exercise 15) shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Use the plurality-with-elimination method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

36. Table 1-36 (see Exercise 16) shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Use the plurality-with-elimination method to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

37. Table 1-39 shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). Find the complete ranking of the candidates using the plurality-with-elimination method.

Number of voters	8	7	5	4	3	2
1st	*B*	*C*	*A*	*D*	*A*	*D*
2nd	*E*	*E*	*B*	*C*	*D*	*B*
3rd	*A*	*D*	*C*	*B*	*E*	*C*
4th	*C*	*A*	*D*	*E*	*C*	*A*
5th	*D*	*B*	*E*	*A*	*B*	*E*

■ TABLE 1-39

38. Table 1-40 shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). Find the complete ranking of the candidates using the plurality-with-elimination method.

Number of voters	7	6	5	5	5	5	4	2	1
1st	*D*	*C*	*A*	*C*	*D*	*E*	*B*	*A*	*A*
2nd	*B*	*A*	*B*	*A*	*C*	*A*	*E*	*B*	*C*
3rd	*A*	*E*	*E*	*B*	*A*	*D*	*C*	*D*	*E*
4th	*C*	*B*	*C*	*D*	*E*	*B*	*D*	*E*	*B*
5th	*E*	*D*	*D*	*E*	*B*	*C*	*A*	*C*	*D*

■ TABLE 1-40

Top-Two IRV. *Exercises 39 and 40 refer to a simple variation of the plurality-with-elimination method called top-two IRV. This method works for winner-only elections. Instead of eliminating candidates one at a time, we eliminate all the candidates except the top two in the first round and transfer their votes to the two remaining candidates.*

39. Find the winner of the election given by Table 1-39 using the *top-two IRV* method.

40. Find the winner of the election given by Table 1-40 using the *top-two IRV* method.

1.5 Pairwise Comparisons

41. Table 1-31 (see Exercise 11) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the method of pairwise comparisons to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

42. Table 1-32 (see Exercise 12) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the method of pairwise comparisons to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

43. Table 1-33 (see Exercise 13) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the method of pairwise comparisons to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

44. Table 1-34 (see Exercise 14) shows the preference schedule for an election with four candidates (*A*, *B*, *C*, and *D*). Use the method of pairwise comparisons to

(a) find the winner of the election.

(b) find the complete ranking of the candidates.

45. Table 1-35 (see Exercise 15) shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Find the winner of the election using the method of pairwise comparisons.

46. Table 1-36 (see Exercise 16) shows the preference schedule for an election with five candidates (*A*, *B*, *C*, *D*, and *E*). The number of voters in this election was very large, so the columns of the preference schedule give the percent of voters instead of the number of voters. Find the winner of the election using the method of pairwise comparisons.

47. Table 1-39 (see Exercise 37) shows the preference schedule for an election with 5 candidates. Find the complete ranking of the candidates using the method of pairwise comparisons. (Assume that ties are broken using the results of the pairwise comparisons between the tying candidates.)

48. Table 1-40 (see Exercise 38) shows the preference schedule for an election with 5 candidates. Find the complete ranking of the candidates using the method of pairwise comparisons.

49. An election with five candidates (A, B, C, D, and E) is decided using the method of pairwise comparisons. If B loses two pairwise comparisons, C loses one, D loses one and ties one, and E loses two and ties one,

(a) find how many pairwise comparisons A loses.

(b) find the winner of the election.

50. An election with six candidates (A, B, C, D, E, and F) is decided using the method of pairwise comparisons. If A loses four pairwise comparisons, B and C both lose three, D loses one and ties one, and E loses two and ties one,

(a) find how many pairwise comparisons F loses. (*Hint*: First compute the total number of pairwise comparisons for six candidates.)

(b) find the winner of the election.

1.6 Fairness Criteria

51. Use Table 1-41 to illustrate why the Borda count method violates the Condorcet criterion.

Number of voters	6	2	3
1st	*A*	*B*	*C*
2nd	*B*	*C*	*D*
3rd	*C*	*D*	*B*
4th	*D*	*A*	*A*

■ TABLE 1-41

52. Use Table 1-32 to illustrate why the plurality-with-elimination method violates the Condorcet criterion.

53. Use Table 1-42 to illustrate why the plurality method violates the IIA criterion. (*Hint*: Find the winner, then eliminate F and see what happens.)

Number of voters	49	48	3
1st	*R*	*H*	*F*
2nd	*H*	*S*	*H*
3rd	*F*	*O*	*S*
4th	*O*	*F*	*O*
5th	*S*	*R*	*R*

■ TABLE 1-42

54. Use the Math Club election (Example 1.10) to illustrate why the Borda count method violates the IIA criterion. (*Hint*: Find the winner, then eliminate D and see what happens.)

55. Use Table 1-43 to illustrate why the plurality-with-elimination method violates the IIA criterion. (*Hint*: Find the winner, then eliminate C and see what happens.)

Number of voters	5	5	3	3	3	2
1st	*A*	*C*	*A*	*D*	*B*	*D*
2nd	*B*	*E*	*D*	*C*	*E*	*C*
3rd	*C*	*D*	*B*	*B*	*A*	*B*
4th	*D*	*B*	*C*	*E*	*C*	*A*
5th	*E*	*A*	*E*	*A*	*D*	*E*

■ TABLE 1-43

56. Explain why the method of pairwise comparisons satisfies the majority criterion.

57. Explain why the method of pairwise comparisons satisfies the Condorcet criterion.

58. Explain why the plurality method satisfies the monotonicity criterion.

59. Explain why the Borda count method satisfies the monotonicity criterion.

60. Explain why the method of pairwise comparisons satisfies the monotonicity criterion.

JOGGING

61. Two-candidate elections. Explain why when there are only two candidates, the four voting methods we discussed in this chapter give the same winner and the winner is determined by straight majority. (Assume that there are no ties.)

62. Equivalent Borda count (Variation 1). The following simple variation of the Borda count method is sometimes used: A first place is worth $N-1$ points, second place is worth $N-2$ points, . . ., last place is worth 0 points (where N is the number of candidates). Explain why this variation is equivalent to the original Borda count described in this chapter (i.e., it produces exactly the same election results).

63. Equivalent Borda count (Variation 2). Another commonly used variation of the Borda count method is the following: A first place is worth 1 point, second place is worth 2 points, . . . , last place is worth N points (where N is the number of candidates). The candidate with the fewest points is the winner, second fewest points is second, and so on. Explain why this variation is equivalent to the original Borda count described in this chapter (i.e., it produces exactly the same election results).

64. The average ranking. The average ranking of a candidate is obtained by taking the place of the candidate on each of the ballots, adding these numbers, and dividing by the number of ballots. Explain why the candidate with the best (lowest) average ranking is the Borda winner.

65. **The 2006 Associated Press college football poll.** The AP college football poll is a ranking of the top 25 college football teams in the country and is one of the key polls used for the BCS (Bowl Championship Series). The voters in the AP poll are a group of sportswriters and broadcasters chosen from across the country. The top 25 teams are ranked using a Borda count: each first-place vote is worth 25 points, each second-place vote is worth 24 points, each third-place vote is worth 23 points, and so on. Table 1-44 shows the ranking and total points for each of the top three teams at the end of the 2006 regular season. (The remaining 22 teams are not shown here because they are irrelevant to this exercise.)

Team	Points
1. Ohio State	1625
2. Florida	1529
3. Michigan	1526

■ TABLE 1-44

(a) Given that Ohio State was the unanimous first-place choice of all the voters, find the number of voters that participated in the poll.

(b) Given that all the voters had Florida in either second or third place, find the number of second-place and the number of third-place votes for Florida.

(c) Given that all the voters had Michigan in either second or third place, find the number of second-place and the number of third-place votes for Michigan.

66. **The 2005 National League MVP vote.** Each year the Most Valuable Player of the National League is chosen by a group of 32 sportswriters using a variation of the Borda count method. Table 1-45 shows the results of the 2005 voting for the top three finalists:

Player	1st place	2nd place	3rd place	Total points
Albert Pujols	18	14	0	378
Andruw Jones	13	17	2	351
Derrek Lee	1	1	30	263

Source: Baseball-Reference.com, *www.baseball-reference.com/awards/awards_2005.shtml*

■ TABLE 1-45

Determine how many points are given for each first-, second-, and third-place vote in this election.

67. **The 2003–2004 NBA Rookie of the Year vote.** Each year, a panel of broadcasters and sportswriters selects an NBA rookie of the year using a variation of the Borda count method. Table 1-46 shows the results of the balloting for the top three finalists of the 2003–2004 season.

Player	1st place	2nd place	3rd place	Total points
LeBron James	78	39	1	508
Carmelo Anthony	40	76	2	430
Dwayne Wade	0	3	108	117

Source: InsideHoops.com, *www.insidehoops.com/lebron-wins-042004.shtml*

■ TABLE 1-46

Determine how many points are given for each first-, second-, and third-place vote in this election.

68. Top-two IRV is a variation of the plurality-with-elimination method in which all the candidates except the top two are eliminated in the first round (see Exercises 39 and 40).

(a) Use the Math Club election to show that top-two IRV can produce a different outcome than plurality-with-elimination.

(b) Give an example that illustrates why top-two IRV violates the monotonicity criterion.

(c) Give an example that illustrates why top-two IRV violates the Condorcet criterion.

69. **The Coombs method.** This method is just like the plurality-with-elimination method except that in each round we eliminate the candidate with the *largest number of last-place votes* (instead of the one with the fewest first-place votes).

(a) Find the winner of the Math Club election using the Coombs method.

(b) Give an example that illustrates why the Coombs method violates the Condorcet criterion.

(c) Give an example that illustrates why the Coombs method violates the monotonicity criterion.

70. **Bucklin voting.** (This method was used in the early part of the 20th century to determine winners of many elections for political office in the United States.) The method proceeds in rounds. **Round 1**: Count first-place votes only. If a candidate has a majority of the first-place votes, that candidate wins. Otherwise, go to the next round. **Round 2**: Count *first- and second-place* votes only. If there are any candidates with a majority of votes, the candidate with the most votes wins. Otherwise, go to the next round. **Round 3**: Count *first-, second-, and third-place* votes only. If there are any candidates with a majority of votes, the candidate with the most votes wins. Otherwise, go to the next round. Repeat for as many rounds as necessary.

(a) Find the winner of the Math Club election using the Bucklin method.

(b) Give an example that illustrates why the Bucklin method violates the Condorcet criterion.

(c) Explain why the Bucklin method satisfies the monotonicity criterion.

RUNNING

71. The Pareto criterion. The following fairness criterion was proposed by Italian economist Vilfredo Pareto (1848–1923): *If every voter prefers candidate X to candidate Y, then X should be ranked above Y.*

(a) Explain why the Borda count method satisfies the Pareto criterion.

(b) Explain why the pairwise-comparisons method satisfies the Pareto criterion.

72. The Condorcet loser criterion. *If there is a candidate who loses in a one-to-one comparison to each of the other candidates, then that candidate should not be the winner of the election.* (This fairness criterion is a sort of mirror image of the regular Condorcet criterion.)

(a) Give an example that illustrates why the plurality method violates the Condorcet loser criterion.

(b) Give an example that illustrates why the plurality-with-elimination method violates the Condorcet loser criterion.

(c) Explain why the Borda count method satisfies the Condorcet loser criterion.

73. Consider the following fairness criterion: *If a majority of the voters have candidate X ranked last, then candidate X should not be a winner of the election.*

(a) Give an example to illustrate why the plurality method violates this criterion.

(b) Give an example to illustrate why the plurality-with-elimination method violates this criterion.

(c) Explain why the method of pairwise comparisons satisfies this criterion.

(d) Explain why the Borda count method satisfies this criterion.

74. Suppose that the following was proposed as a fairness criterion: *If a majority of the voters rank X above Y, then the results of the election should have X ranked above Y.* Give an example to illustrate why all four voting methods discussed in the chapter can violate this criterion. (*Hint:* Consider an example with no Condorcet candidate.)

75. Consider a variation of the Borda count method in which a first-place vote in an election with N candidates is worth F points (where $F > N$) and all other places in the ballot are the same as in the ordinary Borda count: $N - 1$ points for second place, $N - 2$ points for third place, . . . , 1 point for last place. By choosing F large enough, we can make this variation of the Borda count method satisfy the majority criterion. Find the smallest value of F (expressed in terms of N) for which this happens.

PROJECTS AND PAPERS

1 Ballots, Ballots, Ballots!

In this chapter we discussed elections in which the voters cast their votes by means of preference ballots. There are many other types of ballots used in real-life elections, ranging from the simple (winner only) to the exotic (each voter has a fixed number of points to divide among the candidates any way he or she sees fit). In this project you are to research other types of ballots; how, where, and when they are used; and what are the arguments for and against their use.

2 Instant Runoff Voting

Imagine that you are a political activist in your community. The city council is having hearings to decide if the *method of instant runoff voting* (plurality-with-elimination) should be adopted in your city. Stake out a position for or against instant runoff voting and prepare a brief to present to the city council that justifies that position. To make an effective case, your argument should include mathematical, economic, political, and social considerations. (Remember that your city council members are not as well versed as you are in the mathematical aspects of elections. Part of your job is to educate them.)

3 The 2000 Presidential Election and the Florida Vote

The unusual circumstances surrounding the 2000 presidential election and the Florida vote are a low point in American electoral history. Write an analysis paper on the 2000 Florida vote, paying particular attention to what went wrong and how a similar situation can be prevented in the future. You should touch on technology issues (outdated and inaccurate vote-tallying methods and equipment, poorly designed ballots, etc.), political issues (the two-party system, the Electoral College, etc.), and, as much as possible, on issues related to concepts from this chapter (can presidential elections be improved by changing to preference ballots, using a different voting method, etc.).

4 Short Story

Write a fictional short story using an election as the backdrop. Weave into the dramatic structure of the story elements and themes from this chapter (fairness, manipulation, monotonicity, and independence of irrelevant alternatives all lend themselves to good drama). Be creative and have fun.