DISCRETE CONFORMALITY AND GRAPH EMBEDDING

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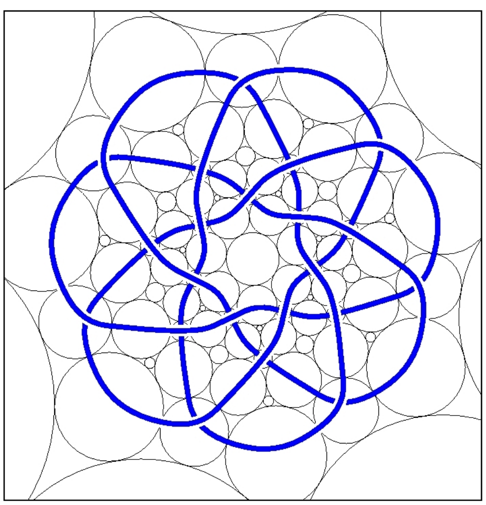
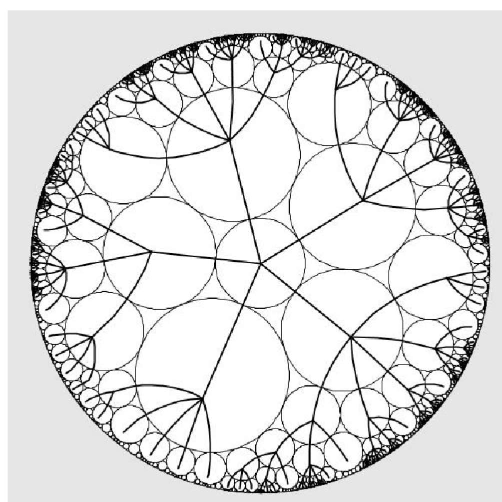
This talk will describe a variety of applications for the topic of "circle packing" in the embedding, manipulation, and visualization of graphs.

Circle packings are configurations of circles having specified patterns of tangency. They arose in a conformal geometry context with Thurston's conjecture and the subsequent proof by Rodin and Sullivan that one could approximate classical conformal mappings of plane domains using maps between circle packings. From that start the topic has blossomed into a fully fledged theory of discrete conformal geometry. For background you can see "Introduction to Circle Packing: the Theory of Discrete Analytic Functions", Cambridge (2005).

We approach the topic from the combinatorial side, however. Suppose you have a locally planar graph G, as might arise in any of numerous applications. Augment G to a triangulation graph T and compute a circle packing P associated with T. The layout of P provides an embedding (or immersion) of G in a geometric setting: the sphere, euclidean plane, hyperbolic plane, or a Riemann surface, as appropriate.





Embeddings of this type have much to recommend them: they are relatively easy to compute (even for large graphs), they preserve underlying symmetries and self-similarities, they are easy to manipulate with boundary values and various other geometric parameters, and they often give natural and visually appealing images. Their main advantage, however, may lie with a certain internal rigidity. This rigidity is strongly conformal in nature, bringing to discrete settings some of the richness of classical conformal geometry: notions of extremal length, harmonic measure, conformal structure, and random walks, for example.

This talk will be largely visual and will illustrate what circle packing might bring to several applications in graph embedding: brain imaging, optimal point distributions, hyperbolic tree embeddings, dual embeddings, random triangulations, and the geometry graphene (2D carbon sheets).

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