Research Statement

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At a quick glance the classical fields of algebraic geometry and symplectic topology seem far apart. The category of algebraic (and more generally holomorphic) functions does not contain any (non-trivial) partitions of unity, which play a fundamental role in topology. It is thus not too surprising that these fields generally evolved in their own separate ways throughout much of their long history. This changed dramatically following Gromov's work [9] on pseudo-holomorphic curves in symplectic topology, which beautifully combined the rigidity of algebraic category with the flexibility of the smooth category, and Witten's work [30] on σ-models in physics, which foretold an incredible algebraic structure for counts of such curves. Since then much of symplectic topology and algebraic geometry has thrived on the newly discovered interplay between the two fields and predictions generated by the blossoming field of string theory. Some of these predictions have been verified mathematically, providing the only "experimental" evidence for string theory and generating much excitement in mathematics. Many other predictions of string theory remain unsolved and are now among the most prominent problems in the two fields.

My research involves the study of pseudo-holomorphic curves in symplectic topology and algebraic geometry and is often motivated by insights generated by string theory. It has two general directions:

(1) analytic studies of deformation properties of pseudo-holomorphic maps from Riemann surfaces to (almost) complex manifolds, for an arbitrary complex structure on the target as well as for a generic one, with an eye toward very different applications in symplectic topology, algebraic geometry, and string theory;

(2) applications of recently developed machinery for explicit computations of Gromov-Witten invariants of complete intersections with the aim of confirming predictions of string theory and going beyond them.

My recent, current, and future work in these directions is outlined in the next two sections.

1 Analytic Methods

A symplectic form on a $2n$-dimensional manifold X is a closed 2-form on X such that ω^n is a volume form on X. A tame almost complex structure on a symplectic manifold (X,ω) is a bundle endomorphism

$$J\colon TX \longrightarrow TX \qquad \text{s.t.} \qquad J^2=-\mathrm{Id}, \quad \omega(v,Jv)>0 \ \ \forall\, v\in T_xX,\ x\in X,\ v\neq 0.$$

If Σ is a (possibly nodal) Riemann surface with complex structure $\mathfrak{j}$, a smooth map $u\colon \Sigma \longrightarrow X$ is called J-holomorphic if it solves the Cauchy-Riemann equation corresponding to $(J,\mathfrak{j})$:

$$\bar\partial_J u \equiv \frac{1}{2}\big(du + J\circ du\circ \mathfrak{j}\big)=0.$$

The image of such a map in X is called a J-holomorphic curve. Holomorphic curves have played an essential role in algebraic geometry throughout its existence. Their emergence as a central theme in symplectic topology has led to interplay between the two subjects, connections with string theory, and heavy use of analytic techniques. This is reflected in the problems described below.

One of the most fundamental objects in Gromov-Witten theory is the moduli space $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ of stable genus g J-holomorphic maps with k marked points in the homology class $A\in H_2(X)$. This compact space may be highly singular, but still determines a rational homology class, called virtual fundamental class (VFC), which is independent of J; integration of cohomology classes against this VFC gives rise to Gromov-Witten invariants. While $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ is often called a "compactification" of its subspace

$$\mathfrak{M}^0_{g,k}(X,A;J)\subset\overline{\mathfrak{M}}_{g,k}(X,A;J)$$

of maps from smooth domains, $\mathfrak{M}^0_{g,k}(X,A;J)$ usually is not a dense (open) subset of $\overline{\mathfrak{M}}_{g,k}(X,A;J)$. This brings up the question whether one can do better.

Problem 1A *Is there a proper natural closed subset* $\overline{\mathfrak{M}}^0_{g,k}(X,A;J)$ *of* $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ *containing* $\mathfrak{M}^0_{g,k}(X,A;J)$*?*

The "natural" requirement includes existence of a VFC for $\overline{\mathfrak{M}}^0_{g,k}(X,A;J)$, which is independent of J and satisfies other geometric properties. If (X,J) is the standard complex projective space $\mathbb{P}^n$, this closed subset should simply be the closure of $\mathfrak{M}^0_{g,k}(X,A;J)$ in $\overline{\mathfrak{M}}_{g,k}(X,A;J)$, but in general such a collection of compactifications is not natural (in particular, it does not respect embeddings $Y\longrightarrow X$).

It is known that the answer to Problem 1A is *no* if $g=0$. On the other hand, it has been speculated since the early days of GW-theory that the answer to Problem 1A is *yes* if $g\geq1$. I confirmed this for $g=1$ in [31, 33], constructing *reduced* genus 1 GW-invariants. The same methods directly carry over to $g>1$, but the resulting affirmative answer to Problem 1A is unsatisfactory because it is not sharp and the resulting reduced GW-invariants include lower-genus contributions.

While Problem 1A concerns a basic issue in GW-theory (and thus is of interest in itself), a satisfactory answer to this problem appears to be key to relating GW-invariants of a compact symplectic submanifold (Y,ω) of a compact symplectic manifold (X,ω) given as the zero set of a transverse bundle section to the GW-invariants of the ambient symplectic manifold X. If $\pi_{\mathcal{L}}:\mathcal{L}\longrightarrow X$ is a holomorphic vector bundle, there is a natural projection map

$$\tilde\pi_{\mathcal{L}}\colon\mathcal{V}^A_{g,k}(\mathcal{L})\equiv\overline{\mathfrak{M}}_{g,k}(\mathcal{L},A;J)\longrightarrow\overline{\mathfrak{M}}_{g,k}(X,A;J),\quad[\tilde u:\Sigma\longrightarrow\mathcal{L}]\longrightarrow[\pi\circ\tilde u:\Sigma\longrightarrow X];\tag{1}$$

the fiber of $\tilde\pi_{\mathcal{L}}$ over an element $[u:\Sigma\longrightarrow X]$ is $H^0(\Sigma;u^*\mathcal{L})$, the space of holomorphic sections of the holomorphic bundle $u^*\mathcal{L}\longrightarrow\Sigma$. If X and $\mathcal{L}$ are sufficiently positive (such as $\mathbb{P}^n$ and sum of positive line bundles) and $g=0$, $\tilde\pi_{\mathcal{L}}$ is in fact a vector bundle and

$$\iota_*[\overline{\mathfrak{M}}_{0,k}(Y,A;J)]^{vir}=e\big(\mathcal{V}^A_{0,k}(\mathcal{L})\big)\cap[\overline{\mathfrak{M}}_{0,k}(X,A;J)]^{vir}.\tag{2}$$

While $\tilde\pi_{\mathcal{L}}$ is not even a vector bundle for $g\geq1$ (even for sufficiently positive X and $\mathcal{L}$), it is shown in [32, 16] that the restriction

$$\tilde\pi_{\mathcal{L}}\colon\mathcal{V}^A_{1,k}(\mathcal{L})\big|_{\overline{\mathfrak{M}}^0_{1,k}(X,A;J)}\longrightarrow\overline{\mathfrak{M}}^0_{1,k}(X,A;J)\tag{3}$$

carries a well-defined euler class, which in turn relates the reduced genus 1 GW-invariants of the submanifold and the ambient manifold:

$$\iota_*[\overline{\mathfrak{M}}^0_{1,k}(Y,A;J)]^{vir}=\mathrm{PD}_{[\overline{\mathfrak{M}}^0_{1,k}(X,A;J)]}e\big(\mathcal{V}^A_{1,k}(\mathcal{L})\big).\tag{4}$$

A satisfactory affirmative answer to Problem 1A for $g\geq2$ should lead to an affirmative answer to the following problem for the same g.

Problem 1B *If (X,J) is a sufficiently regular Kahler manifold and $\mathcal{L}\longrightarrow X$ is sufficiently positive, is there an analogue of the relation (4) for $g\geq 2$?*

Affirmative answers to Problems 1A and 1B for any given genus g would provide means for computing GW-invariants of complete intersections in $\mathbb{P}^n$ (and in other smooth toric varieties) using localization by a group action on the ambient space, since the standard and reduced genus g GW-invariants would differ only by lower-genus invariants. This computation would be made more efficient if the genus g analogue of the cone (3) is desingularized in a natural way (turning it into a vector orbi-bundle over an orbifold), so that Atiyah-Bott's classical localization theorem [1] could be applied directly.

Problem 1C *If (X,J) is $(\mathbb{P}^n,J_0)$ or another toric variety and $\mathcal{L}\longrightarrow X$ is sufficiently positive, does the genus g analogue of (3) admit a natural desingularization?*

For $g=1$, this is accomplished in [27]. Once Problems 1A-1C are completed for any given genus $g\geq 2$, it should be fairly straightforward at this point to obtain a closed formula for a generating function for the genus g GW-invariants; see Section 2.

While Problems 1A-1C appear rather different, they mostly reduce to the same issue: analyzing obstructions to smoothing a J-holomorphic map (into Y, X, or $\mathcal{L}$) from a singular domain. Furthermore, the obstruction space in these cases is fairly well-described by the combinatorial data about the singular map (distribution of the degrees of the map between the irreducible components of the domain). The main hurdle in each genus $g\geq 2$ is thus likely to be Problem 1A.

This past year, I became involved in a joint project that aims to adapt the analytic techniques used in the proof of the BCOV prediction [2] for the closed genus 1 GW-invariants of a quintic threefold to the genus 1 real invariants, i.e. counts of maps commuting with involutions. The aim of this project to define such invariants intrinsically, at least under some topological assumptions on the target, relate such invariants of complete intersections to the invariants of the ambient projective space, and reduce them to the graph sums in [29]. The initial steps in the first part of this project are carried out in [6, 10]. The second and third parts will run [16, 31, 32, 33], with some additional care, in the presence of an anti-symplectic involution on the target. Since the graph sums in [29] are shown in [24] to reduce the mirror formula in [29], this project would resolve the following problem.

Problem 2 *Complete the proof of the prediction of [29] for the real genus 1 GW-invariants of Calabi-Yau complete intersection threefolds.*

The obstruction space can also be described in terms of the combinatorics of the map for the moduli space of relative maps: J-holomorphic maps into X that have a specified contact with a divisor Y. In the case the divisor is smooth, moduli spaces of relative maps

$$\mathfrak{M}^{\rho}_{g,k}(X,Y,A;J)\subset\overline{\mathfrak{M}}_{g,k}(X,A;J)$$

are typically compactified "outside" of $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ by decorating some of the J-holomorphic maps with additional "rubber" structure [12, 14, 15]. However, one might expect a natural compactification of the former space inside of the latter.

Problem 3 *Does the moduli space of relative maps $\mathfrak{M}^{\rho}_{g,k}(X,Y,A;J)$ admit a natural compactification inside of $\overline{\mathfrak{M}}_{g,k}(X,A;J)$?*

If $g=0$, (X,J) is a smooth projective variety, and Y is a smooth very ample hypersurface, this compactification must simply be the closure [7], but in general such a collection of compactifications is not natural (similarly to the situation with Problem 1A). In [36], I give a natural extension of the compactification of [7] to the case of an arbitrary smooth symplectic hypersurface Y in a symplectic manifold X in genus 0. The original motivation for [36] came from Li-Ruan's program of symplectic birational geometry [17], which involves comparing the genus 0 GW-invariants of a symplectic divisor with the genus 0 GW-invariants of the ambient manifold; depending on the resolution of Problems 4B, this program may shed light on Kollár's conjecture on rational connectedness in algebraic geometry (see below). More recently, I came to realize that the approach of [36] should lead to genus 0 GW-invariants relative to a simple-normal-crossings divisor (this problem has been studied for over a decade).

Analysis of obstructions to deforming J-holomorphic maps seems to be also essential to two other, very different, settings: symplectic and algebraic birational geometry and Gopakumar-Vafa integrality predictions for Calabi-Yau manifolds, as described below.

A smooth algebraic manifold X is called uniruled (resp. rationally connected or RC) if there is a rational curve through every point (resp. every pair of points) in X. According to [13, 25], a uniruled algebraic variety admits a non-zero genus 0 GW-invariant with a point insertion (i.e. a count of stable maps in a fixed homology class which pass through a point and some other constraints). This implies that the uniruled property is invariant under symplectic deformations. The RC property is known to be invariant under integrable deformations of the complex structure [13, 25]. It is a conjecture of Kollár that the RC property is invariant under symplectic deformations as well; this was recently confirmed in dimension 3 by Z. Tian [26], building up on [28]. It is still unknown if every RC algebraic manifold admits a nonzero genus 0 GW-invariant with two point insertions; this would immediately imply Kollár's conjecture.

As GW-invariants are symplectic invariants, it is natural to consider the parallel situation in symplectic topology; this may also provide a different approach to Kollár's conjecture. Thus, a symplectic manifold (X,ω) is called *uniruled* (resp. *RC*) if for some ω-compatible almost complex structure J there is a genus 0 connected rational J-holomorphic curve through every point (resp. every pair of points) in X. This leads to the following two pairs of problems.

Problem 4A *Let J be any almost complex structure on a uniruled (resp. RC) compact symplectic manifold (X,ω). Is there a connected rational J-holomorphic curve through every point (resp. every pair of points) in X?*

Problem 4B *Does every uniruled (resp. RC) compact symplectic manifold (X,ω) admit a genus 0 GW-invariant with a point insertion (resp. two point insertions)?*

The affirmative answer to Problem 4B would immediately imply the affirmative answer to Problem 4A. In a way, the affirmative answer to any of the four questions could be viewed as contrary to the spirit of flexibility in symplectic topology; indeed, the uniruled case of the latter problem is known only under the rigidity assumptions that X is either Kahler or admits a Hamiltonian S^1-action [20]. However, there are currently no known counter-examples to the speculations made in these four questions.

If $u\colon \mathbb{P}^1 \longrightarrow X$ is a J-holomorphic curve into a Kahler manifold and for *some* $z\in\mathbb{P}^1$ the evaluation map

$$H^0(\mathbb{P}^1; u^*TX) \longrightarrow T_{u(z)}X, \qquad \xi \longrightarrow \xi(z),$$

is onto, then $H^1(\mathbb{P}^1; u^*TX)=0$, i.e. u is unobstructed. This statement is key to the arguments in the algebraic setting [13, 25], but its natural extension to the non-integrable setting is *false*, even if the evaluation is surjective for *every* $z\in\mathbb{P}^1$. However, its implications may still be true. In particular, for the interplay between openness and closedness of various properties of complex structures exhibited in the proof of deformation invariance of the RC property for integrable complex structures in [13] to extend to the non-integrable complex structure, the vanishing of the obstruction space needs to hold only generically in a family of J-holomorphic maps covering X. This generic vanishing still seems to be quite plausible; studying properties of the linearized operators $\bar{\partial}$-operators will be key to understanding this issue as well. The resulting investigation should uncover properties of J-holomorphic maps that may be of importance to future development of the subject of J-holomorphic curves. It could also be relevant to the following old-standing problem, which is motivated by the Gopakumar-Vafa conjecture on integrality of certain combinations of GW-invariants.

Problem 5 *Is it the case that for a generic almost complex structure J on a Calabi-Yau threefold (X,ω) all J-holomorphic curves in X are isolated (and smooth)?*

A number of claims concerning this problem have been made, but none has worked out. All of the attempts so far have involved variations on standard transversality arguments in symplectic topology, without any attempt to study properties of the linearizations of the $\bar{\partial}$-operator. My aim is to do so, beginning with the simplest possible domain, $\mathbb{P}^1$, when these operators can be written explicitly.

2 Combinatorial methods

Among the most striking mathematical predictions of string theory are the so-called mirror symmetry formulas for GW-invariants of Calabi-Yau 3-folds, especially quintic hypersurfaces in $\mathbb{P}^4$; they relate generating functions for GW-invariants to Kahler properties of moduli spaces of Calabi-Yau 3-folds. The first such prediction [3] expressed the genus 0 GW-invariants of a quintic in terms of holonomy around a singular fiber in the moduli space of "mirror quintics", with the holonomy computed mathematically; this prediction was confirmed in the mid-1990s [8, 18]. The next prediction [2] expressed the genus 1 GW-invariants of a quintic in terms of what is now called the BCOV torsion, a function on the moduli space of "mirror quintics" involving eigenvalues of a family of Laplacians. A mathematical computation of the latter was completed in [5], while the prediction of [2] for GW-invariants was verified in [35]. A mirror formula for the disk invariants of a quintic threefold is obtained in [21]. In [24], such a formula is obtained for the annulus and Klein bottle invariants starting from the graph definition of [29].

The main mathematical approach to the string theory predictions for GW-invariants has been to express the desired GW-invariants of a complete intersection in a toric variety X as an integral over the moduli space of stable maps to X, as in Problem 1B, and then to localize the integral to the fixed loci of a torus action on the moduli space. This leads to combinatorial problems involving summations of rational function in several variables over many different graphs. Many such problems for genus 0 GW-invariants with 1 marked point were handled in the 1990s. More recently, I developed methods to use solutions to these problems to explicitly compute generating functions of genus 0 multi-pointed GW-invariants and genus 1 GW-invariants as transforms of the genus 0 generating functions. So far, these methods have been applied only to projective complete intersections [34, 35, 24, 22, 37]. This leads to the following set of problems which should be approachable by similar means.

Problem 6A *Let X be a toric complete intersection. Describe a transformation that determines*

(a) *a generating function for genus 0 2-point GW-invariants of X in terms of a generating function for 1-point GW-invariants;*

(b) *a generating function for genus 0 n-point GW-invariants of X, with $n \geq 3$, in terms of a generating function for* 1*-point GW-invariants.*

Problem 6B *Let X be a toric complete intersection. Describe transformations that determine generating functions for the closed genus 1, annulus, and Klein bottle n-point GW-invariants of X in terms of a generating function for genus 0 1-point GW-invariants.*

Problem 6C *Let X be a toric complete intersection. Describe a transformation that determines a generating function for genus g GW-invariants of X, with $g \geq 2$, in terms of a generating function for genus 0 1-point GW-invariants.*

Based on the work so far, these transformations should depend only on the cohomology of X and the chern class of the vector bundle "dual" to the complete intersection. Part (a) of Problem 6A is solved in [23]. At this point, there should be no fundamental geometric difficulty in completing Problem 6A. The reason it is broken into two parts is that generating functions for genus 0 n-pointed invariants, with $n \geq 3$, are obtained as sums over trivalent graphs with n marked points from the 1- and 2-marked point generating functions. In the case of a projective complete intersection, the generating functions of Problem 6B can already be obtained by summing over trivalent graphs. Progress for other complete intersections is contingent extending of the $g=1$ cases of Problems 1A-1C and 2 from the projective space to other toric varieties, which seems feasible at least in some cases. Problem 6C is not approachable at this point, even for projective complete intersections, since it requires a satisfactory resolution of Problems 1A-1C for $g \geq 2$ in the same settings.

The primary motivation behind Problems 6B and 6C is to test mirror-symmetry predictions of string theory; Problem 6A with $n=2$ and $n \leq g+1$ is needed as an input. This would be of particular interest due to recent string theory predictions for genus 1 GW-invariants of certain Grassmannian complete intersection Calabi-Yau 3-folds. Even of greater mathematical interest would be a confirmation of mirror symmetry predictions for the GW-invariants of a quintic in genus 2 and higher; string theorists [11] can in principle generate formulas up to genus 51 that relate them to higher-genus BCOV torsion. As the latter is poorly understood in Kahler geometry, a confirmation of these predictions may be of interest in this field as well.

Opening a new direction, [4] provides a mirror formula for the genus 0 stable quotients invariants of Fano and Calabi-Yau complete intersections. These invariants, defined in [19], arise from smaller moduli spaces than GW-invariants and give a simpler mirror formula. The argument is an intricate twist on Givental's proof of mirror symmetry for Gromov-Witten invariants and involves bootstrapping from the Fano to the Calabi-Yau cases and flipping the use of the two variations of Givental's approach in the Fano and Calabi-Yau cases. The conclusion of [4] suggests that the stable-quotient invariants are somehow closer to the B-side of mirror symmetry than the GW-invariants and makes it possible to compute the genus 1 stable invariants of Fano and Calabi-Yau complete intersections, at least with some marked points (currently in preparation). There is in fact a sequence of intermediate invariants between the GW- and SQ-invariants; studying the wall-crossing phenomenon through this sequence may lead to the proof of the integrality of the mirror transform map predicted by string theorists.

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