# Editors' Note on
# "The symplectic sum formula for
# Gromov-Witten invariants"

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The authors have acknowledged the following errors brought to their attention by Tehrani and Zinger [TZ]:

(i) Parts of the analysis in Sections 6-8 address only the basic case of maps with intersection multiplicity $s = 1$.

(ii) Formula (7.5) for the adjoint $\mathbf{D}_F^*$ is not correct.

(iii) The sign of the curvature in (8.7) is wrong, invalidating the proof of Proposition 8.2.

The Editors note that these errors invalidate the proof of the main result, the Symplectic Sum Theorem of [IP1]. Further, while the statement of the main result asserts a topological formula for symplectic manifolds, the proof requires the analytic hypothesis that the relevant moduli spaces are "generically admissible."

The authors have posted a Corrigendum [IP2] to the paper on the arXiv: http://arXiv.org/abs/1510.06943.

## References

[IP1] E.-N. IONEL and T. H. PARKER, The symplectic sum formula for Gromov-Witten invariants, *Ann. of Math.* **159** (2004), 935–1025. MR 2113018. Zbl 1075. 53092. http://dx.doi.org/10.4007/annals.2004.159.935.

[IP2] E.-N. IONEL and T. H. PARKER, Corrigendum: The symplectic sum formula for Gromov-Witten invariants, 2015. arXiv 1510.06943.

[TZ] M. F. TEHRANI and A. ZINGER, On symplectic sum formulas in Gromov-Witten theory. arXiv 1404.1898v2.

# Some Comments on the Annals Editors' Official Statement
Aleksey Zinger              Updated: 12/05/16

I appreciate the Annals Editors taking the time to issue a statement concerning [IP5], in contrast to the Inventiones Editors continuing to knowingly ignore the situation with [LR]. Unfortunately, the Annals note significantly understates the problems with [IP4, IP5] and acknowledges only part of common knowledge regarding these papers in the field; see below. Even this very partial acknowledgment further substantiates that the $655k 2009 NSF grant (an equivalent of 5-6 junior grants) to E. Ionel after 5+ years of no completed work (as arXiv shows) was based exclusively on hype, which is so typical of the field of symplectic topology. Following this statement, one might have hoped Ionel-Parker would have the minimal decency and sense to retract [IP4, IP5] instead of dragging this out further and continuing to damage the field and especially the hardly any junior people who dare to enter it; this has not happened. Some "leaders" in the field, such as D. McDuff, Y. Eliashberg, and T. Mrowka, also appear to have no desire to move on, perhaps to cover up their own poor judgment regarding E. Ionel's work in the past.

The errors listed by the Editors concern highly technical, but central, issues that had not been known before [FZ]; they are summarized as Issue C on page 4 of [Z1]. Ionel-Parker acknowledge that what had been described in [IP5] as the most important part was in fact completely wrong. The correct statements of the main theorems of [IP5] had appeared in [LR] almost 3 years earlier (the brief announcement [IP3] appeared on arXiv 3 months after [LR] and was clearly intended just to lay a claim to the symplectic sum formula and prevent others from working on its proof).

The *further* sentence in the Editors' note refers to the most well-known issue with [IP4, IP5], Issue A on page 4 of [Z1]: these papers claim to construct relative GW-invariants and to prove a symplectic sum formula without any semi-positivity assumption. While there is no mention of a semi-positive restriction in the 4-page summary of [IP4], the 7-page summary of [IP5], or any of the main theorems in [IP5], the rest of the papers considers only semi-positive cases whenever GW-invariants are concerned. This issue is even more severe with [IP4] as Remark 1.9 in [IP4] claims that their methods apply to the general case because *in a separate paper [IP5]* they *describe* an alternative VFC construction. This *paper* is listed as *in preparation* in the second (09/01) and third (01/04) arXiv versions of [IP4]. This remark replaced Remark 1.8 in the first version of [IP4], which claimed that the semi-positive restriction can be removed because of the VFC construction of [LT]. However, applying this construction would have required gluing maps with components into the "rubber", which is not done in [IP4]. The VFC construction advertised in [IP4] is claimed in [IP6], which appeared on arXiv 11.5 years after the second arXiv version of [IP4]. It builds on [CM], which first appeared on arXiv almost 5.5 years after the second version of [IP4]. For the most crucial analytic points in the first two versions of [IP6], which require gluing maps with components into the rubber, the reader is referred back to [IP4, IP5]; these two papers restrict to "semi-positive" cases precisely to avoid such gluing and Ionel-Parker had of course known this.

An even more fundamental issue with [IP5] than Issues A and C is Issue B on page 4 of [Z1]. As explained in [Z1] in response to Ionel-Parker's defense of their infamous $S$-matrix, its presence implies that [IP5] did not establish even compactness for maps into a symplectic sum fibration; this is absolutely fundamental to the aims of [IP5] as Ionel-Parker themselves state. The $S$-matrix appears in [IP5] because the authors do not take the quotient by the $\mathbb{C}^*$-action on the "non-trivial"

maps into the so-called "rubber" (this quotient is taken in [LR, J.L] and the "trivial maps" do not even appear in [LR, J.L]). The compactness conclusions stated in [IP5] imply that sequences of maps have countable collections of positive-dimensional families of limits, contrary to what is needed according to the authors. The positive part gives rise to the $S$-matrix; Y. Eliashberg told me in 2012 that he had long known from symplectic field theory considerations [EGH] that the $S$-matrix should not appear and that he had tried to convince E. Ionel of this several times. The countable part gives rise to the inclusion-exclusion argument in [IP5], which perplexed D. McDuff before 2004. Ionel-Parker themselves omit the $S$-matrix in the "proof" of Theorem 11.1 in [IP6] without any comment, in spite of their insistence in the response to [FZ] that this cannot be done.

While Ionel-Parker have at least issued [IP7] admitting some problems with [IP5], the Li-Ruan approach to the issues raised has consisted of issuing insult-filled [LR14] containing no specific contradiction to any objection raised to [LR]. As explained in [Z2], [LR14] makes it clear that [LR] did not provide even a suitable setup for a proof of the symplectic sum formula and that Li-Ruan believe that invariants arising from a moduli space depend only on its virtually main stratum (and not on the compactification). Meanwhile, Y. Ruan collected NSF RTG and FRG grants in addition to a huge personal grant and has been spending most of his non-teaching time at BICMR to schmooze up his way into the Chinese Academy of Sciences after three decades in the US.

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[IP4] E. Ionel and T. Parker, *Relative Gromov-Witten invariants*, Ann. of Math. 157 (2003), no. 1, 45–96

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[IP6] E. Ionel and T. Parker, *A natural Gromov-Witten virtual fundamental class*, math/1302.3472

[IP7] E. Ionel and T. Parker, *Corrigendum: the symplectic sum formula for Gromov-Witten invariants*, math/1510.06943

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[Z1] A. Zinger, *Comments on Ionel-Parker's response*, available on the author's website

[Z2] A. Zinger, *More on the Li-Ruan situation*, available on the author's website