MAT 615: Complex Curves and Surfaces
Spring 2009

Problem Set 2
Due on Tuesday, 03/03, at 12:40pm

Please write up concise solutions to 3 problems; about half a page for each problem should do.

Problem 1 (5 pts)

Let S be a compact connected surface of genus g and $p_0 \in S$. By the Jacobi inversion theorem, the map

$$S^{(g)} \longrightarrow Jac(S) \equiv H^0(S; \mathcal{K}_S)^* / \Lambda_S, \qquad [p_1, \ldots, p_d] \longrightarrow \sum_{i=1}^{i=g} \int_{p_0}^{p_i} \cdot \,,$$

is onto and generically one-to-one; see p236 for notation. If $g = 1$, it is a biholomorphism (presenting every genus 1 curve as $\mathbb{C}/\Lambda$). Describe this map in the case $g = 2$.

Problem 2 (5 pts)

Describe all special divisors on a smooth compact Riemann surface of genus 0,1 and 2.

Problem 3 (5 pts)

Let $C, D_1, D_2 \subset \mathbb{P}^2$ be smooth cubics. If

$$C \cdot D_1 = \sum_{i=1}^{i=9} p_i$$

as divisors on C and D_2 passes through $p_1, \ldots, p_8$, then $p_9 \in D_2$.

Problem 4 (5 pts)

Let $C \subset \mathbb{P}^n$ with $n \geq 3$ be a smooth (connected) curve of genus 1 and degree 4. Show that C is contained in some linearly embedded $\mathbb{P}^3 \subset \mathbb{P}^n$ and is the intersection of two quadric (degree 2) surfaces in that $\mathbb{P}^3$.