MAT 615: Complex Curves and Surfaces

Problem Set 1
Written Solutions (if any) due by Wednesday, 02/08, 9:45am

Please figure out all of the problems below and discuss them with others.

If you have not passed the orals yet, *you are encouraged to write up concise solutions to problems adding up to 10 points in total.*

Problem 1 (5 pts)

Let $C \subset \mathbb{P}^n$, with $n \geq 3$, be a any curve. Show that there exists a point $p \in \mathbb{P}^n$ which is not contained on any line in $\mathbb{P}^n$ meeting C in at least 3 points. (this is related to p215 bottom)

Problem 2 (5 pts)

How does the second statement of Abel's theorem on p227 imply the first?

Problem 3 (5 pts)

The period matrix Ω on p228 is the matrix of a certain natural homomorphism with respect to certain bases. What are these?

Problem 4 (5 pts)

Show that $\mathbb{P}^{1(d)} = \mathbb{P}^d$ (see p236).

Problem 5 (5 pts)

Let S be a compact connected surface of genus g and $p_0 \in S$. By the Jacobi inversion theorem, the map

$$S^{(g)} \longrightarrow Jac(S) \equiv H^0(S; \mathcal{K}_S)^*/\Lambda_S, \qquad [p_1, \ldots, p_d] \longrightarrow \sum_{i=1}^{i=g} \int_{p_0}^{p_i} \cdot \, ,$$

is onto and generically one-to-one; see p236 for notation. If $g = 1$, it is a biholomorphism (presenting every genus 1 curve as $\mathbb{C}/\Lambda$). Describe this map in the case $g = 2$.