MAT 615: Complex Curves and Surfaces
Spring 2023

Course Information

Course Instructor

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Course Website

All homework assignments and various updates will be posted on the course website,

`http://math.stonybrook.edu/~azinger/mat615-spr23`.

Please visit this website regularly.

About the course

Contrary to what the official title might suggest, the focus of this course will be on fairly classical topics in complex geometry that come up in a range of modern areas of geometry. Thus, this will be more of an intermediate-level, second-year style course, rather than an advanced, topics-style course. However, there may be some digressions into current topics related to Gromov-Witten theory broadly construed. You should have a firm grasp of the material covered in the core graduate courses (especially 531 and 536) and be familiar with foundations of complex geometry (main statements and concepts from Chapters 0 and 1 of Griffiths and Harris). The enrollment in this course is limited to the PhD students in mathematics who have passed their comps.

Grading

Your grade will be based on class participation in all possible forms, including in discussions in class, in office hours, and at other times with your classmates, and by handing in written solutions to some homework problems. As there is not much point of taking a fairly general math course without doing some practice problems, please work out and discuss with others all homework problems. These will generally be in the style of typical orals questions. *If you have not passed the orals yet*, you are encouraged to hand in solutions to some problems (as indicated on each assignment) by the deadline; this would be a rather nominal burden. *If you have passed the orals already*, please help others with the homework problems (as part of your class participation).

Textbook

The main textbook is *Principles of Algebraic Geometry* by Phillip Griffiths and Joseph Harris; we will cover parts of Chapters 2 and 4. While the textbook is required for the course, the bookstore may not stock it; please acquire it ahead of time elsewhere (e.g. at amazon which usually sells it at 20% off the list price with free shipping). We will also go through Richard Hind's *Lectures on Moduli Spaces of Elliptic Curves*, math/0812.1803. There may be some occasional material from other books as well, which will hopefully be available at the library; I will try to cover any such material thoroughly in class.

Course Description

Complex manifolds play a prominent role throughout geometry. In addition to being fundamental to algebraic geometry, many areas of differential geometry, and string theory, they provide a wealth of concrete example in such fields as symplectic topology and topology of 4-manifolds. The most common modern approach to complex manifolds is heavily algebraic, such as in Robin Hartshorne's *Algebraic Geometry*. In contrast, the approach taken in *Principles of Algebraic Geometry* is geometric and thus analytic. The aim of this course is to apply the general statements and techniques developed in Chapters 0 and 1 of the book to study complex curves and surfaces (complex manifolds of complex dimension 1 and 2). Here is a tentative list of topics:

1. Foundations of complex curve theory. G&H, Chapter 2, Sections 1-3.
2. Moduli space of elliptic curves. Hain's notes.
3. Grassmannians of 2- and 3-planes and enumerative geometry of low-degree curves. G&H, Chapter 1, Section 5. (This part will likely be omitted due to time constraints.)
4. Foundations of complex surface theory, Enriques-Kodaira "classification". G&H Chapter 4.

Homework Assignments

If you want to learn this material, it is essential to work thoroughly through the text and do as many exercises as possible. While the book itself contains no exercises, I will distribute problem sets every two weeks or so. If you have not passed your orals exam, you are encouraged to write up solutions to a subset of problems (as specified in each assignment) by the due date; *late problem sets will not be accepted*. In addition to the problems you write up, you should figure out all of the remaining exercises for yourself. Feel free to discuss any of the exercises with anyone else, but do write your own solutions.