Errors and Typos in Griffiths&Harris

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Errors/Omissions

p5, $\bar\partial$-Poincaré Lemma in One Variable: the statement and proof apply with the open disk $\Delta\subset\mathbb{C}$ replaced by any bounded open subset. The annulus case is needed to establish the extension of the $\bar\partial$-Poincaré Lemma stated in the middle of p27.

p35, #2,3: the completeness conditions for a sheaf need to be stated for an infinite cover. The book's definition does not imply the infinite-cover condition even for sheaves over $\mathbb{Z}\subset\mathbb{R}$. Without the infinite-cover condition, $\check{H}^0$ need not be the space of global sections.

p104, Lemma: the proof is completely wrong. It is based on the premise that a linear subspace W of an inner-product product space V is dense in V if and only if the orthogonal complement of W in V is 0. The "only if" is of course true. The "if" part is true if V is complete. It need not be true if V is not complete, an example is in Remark on p10 of

http://www.math.stonybrook.edu/∼azinger/mat531-spr11/hw10/ps10sol.pdf

p139, middle: the definition of $c_1(L)$ in H^2_{DR} is off by sign. It implicitly uses an identification between Čech and de Rham cohomologies. The only such identification described in the book is at the bottom of p44. This identification differs by $(-1)^{p(p-1)/2}$ on the p-level from the identification induced via the double complex

$$\big(\check{C}^p(\mathfrak{U},\mathcal{A}^q), D_{p,q}\equiv\delta+(-1)^p d\big).$$

The latter is the "natural" identification of $\check{H}^2$ and H^2_{DR} for the purposes of defining $c_1(L)$ in the de Rham cohomology, so that both statements in Proposition on p141 hold. The proof of this proposition contains another sign error on p141 (which cancels the sign error in the definition of $c_1(L)$ in the de Rham cohomology): the 3rd and 4th displayed equations in the proof reverse the relation between θ_α and θ_β *worked out in Section 5 Chapter 1* (bottom of p72). The 4th equation is off by sign even from the last equation on the followig page. Once the latter sign error is fixed, one gets -1 for $\int_{\mathbb{P}^1} c_1(\mathcal{O}(1))$ with the book's definition of $c_1(L)$ in the de Rham cohomology.

p126, middle: the specialization of the general index statement deduced from the unproved Hodge-Riemann bilinear relations to Kähler surfaces is precisely the statement obtained in the top third of p125.

p195, 2nd displayed eqn: need to add $\dim(\Lambda\cap V_{n-k+i-a_i-1})=i-1$; this is used when *choosing a special basis* in the bottom half of p195

p488, top: the proof is missing the a priori possibility of torsion in $H^2(M;\mathbb{Z})$, but this can be taken into account

p508 bottom half, p510 top: the Euler characteristic inequality on p510 requires the additional assumption that a generic fiber C of f is connected. For example, if $\pi\colon \mathbb{S}_k\longrightarrow\mathbb{P}^1$ is a Hirzerbruch surface and $f:\mathbb{S}_k\longrightarrow\mathbb{P}^1$ is the composition of π with a double cover $\mathbb{P}^1\longrightarrow\mathbb{P}^1$, then C is the disjoint union of two copies of $\mathbb{P}^1$ and so $\chi(\mathbb{P}^1)\chi(C)=8$, while $\chi(\mathbb{S}_k)=4$. The lemma on p505, which corresponds to the $g(C)$ vs. $\pi(C)$ inequality on p508, is proved only for reduced curves. For the purposes of the Euler characteristic inequality on p510, which is used in particular in the middle and bottom of p557, one needs to consider non-reduced curves.

p511, lines 16,17: *a holomorphic map one-to-one away from a finite a collection of points of N* is weaker than a holomorphic birational map $\pi\colon M\longrightarrow N$ unless M is assumed to be connected.

p514, middle: the argument depends on $\iota\colon S\longrightarrow\mathbb{P}^1$ being a submersion

p521, bottom half: this argument is wrong. The 4-line displayed equation should end with

$$\frac{1}{2}\sum_{\nu\neq\nu'}a_{\nu'}C_\nu\cdot C_{\nu'}-\left(\sum_{\nu=1}^{k}a_\nu-1\right).$$

The reasoning just below does not imply this expression is nonnegative if $m_\nu>1$. On the other hand, the pencil $\{C_\lambda\}_{\lambda\in\mathbb{P}^1}$ on S can be replaced by a pencil $\{\widetilde{C}_\lambda\}_{\lambda\in\mathbb{P}^1}$ on a blowup $\pi\colon\widetilde{S}\longrightarrow S$ of S so that all the curves $\widetilde{C}_\lambda$ are disjoint (as in the proof of (1) on pp510/1). By the proof Noether's Lemma on p513, the map

$$\pi\colon\widetilde{S}\longrightarrow\mathbb{P}^1,\qquad \widetilde{C}_\lambda\in x\longrightarrow\lambda\in\mathbb{P}^1,$$

is the composition of a blowdown $S\longrightarrow\mathbb{S}_k$ and projection $\mathbb{S}_k\longrightarrow\mathbb{P}^1$ for some $k\in\mathbb{Z}^{\geq 0}$. This implies that every irreducible component of every curve $\widetilde{C}_\lambda$ is isomorphic to $\mathbb{P}^1$. Since the points of the base locus are smooth points of every C_λ, the same conclusion holds for every curve C_λ.

p557, bottom: the treatment of the $q=1$ case either depends on knowing that the fibers of the Albanese map in this case are connected (which has not been shown) or factoring through a covering of its target as done in the bottom half of p556 and at the top of p557.

p580, middle: it is also needed that $G_\lambda\cap G_{\lambda'}=\mathit{emptyset}$ for $\lambda\neq\lambda'$. This can be achieved by removing the base locus from all curves in the pencil.

Typos

p16, lines 9,10: need *regular* covering
p16, line -2: *local* antiholomorphic functions
p27, line -5: the last denominator is $\partial\bar{z}_j$
p40, line above Basic Fact: $\delta^*\sigma=\mu$
p63, line -4: *compact* analytic subvarieties
p64, line 11: *compact* analytic subvariety
p65, lines 7,10,12: $\varphi\longrightarrow\varphi^{-1}$
p65, line 14: $a_0+a_1x_1+\ldots+a_nx_n\longrightarrow a_{0,1}+a_{1,1}x_1+\ldots+a_{n,i}x_n$
p77, line 4: $\theta^*\longrightarrow\theta$
p78, middle, above θ_E matrix: which lemma?
p78, middle, θ_E matrix: (1,2)-entry should be $-{}^t\bar{A}$
p78, middle, Θ_E matrix: the term in (1,1)- and (2,2)-entries should have +

p78, next display: last terms come with $-$ signs; the identities hold only after the projections

p85, bottom displayed expression: first lines missing $\sum_{\xi,\xi'}$

p87, 2nd displayed expression: last exponent of 1/2 should be outside of the square bracket
p105, line 3: $+\bar{\partial}_N^*\bar{\partial}_M^*$
p123, line -12: $n-k=p+q$ (try $p,q{=}0$ and $n{=}2$)
p129, line 3: *begin*
p130, top: f is square free
p134, line -9: $f^*([D])=[f^*(D)]$
p148, Proposition: $\Theta=(2\pi/\sqrt{-1})\omega$
p153, lines 13,14,-1 (twice); p154, lines 3,-6,-2: $\sqrt{-1}/2 \longrightarrow \sqrt{-1}$ (see bottom of p111)
p153, lines -10,-8,-7,-5,-3: second summands are missing $(-1)^{p+q}$
p153, line -3: $\sum_\alpha$
p153, line -1: RHS missing $(-1)^{p+q}$
p154, bottow 2 displayed expressions (6 times);
p155, lines 2,4: $2\sqrt{-1} \longrightarrow \sqrt{-1}$
p155, lines 4,5: $4\pi \longrightarrow 2\pi$

p160, line -5: $-\sqrt{-1}/2 \longrightarrow -\sqrt{-1}$ (see bottom of p111)
p160, lines -3,-2 (3 times); p161 lines 2,3,6,10,11 (7 times): there should be no factor of 2 in front
p160, line -2: $+1/2\sqrt{-1} \longrightarrow -\sqrt{-1}$
p161, line 3: $-1/2\sqrt{-1} \longrightarrow +\sqrt{-1}$
p161, lines 10,11: $4\pi \longrightarrow 2\pi$ (with the above changes)
p162, line 7: missing) before $\neq$
p162, line 11: *a* section
p169, line -5: $\mathbb{P}^{k+1}\supset\mathbb{P}^k$
p170, 1.: smooth *projective*
p180, middle, (*): $\otimes \longrightarrow \oplus$
p188, middle, $g_{ij}=\det J_{ij}=z(i)_j^{-n+1}$

p193, subsection heading: only Definitions here; the other two are in the next two subsections
p195, line 12: equality holds for $\Lambda\in W_{a_1,\ldots,a_k}$
p202, line -14: $b_{\beta-1} \longrightarrow b_\beta-1$
p206, line 2: left-hand row $\longrightarrow$last column
p206, top display: missing $(-1)^d$ in front the last expression
p206, line -10: $(n{+}1)$-planes $\longrightarrow n$-planes

p215, line -12: in Section 4 of Chapter 1 $\longrightarrow$on page 173 (this is in Section 3 of Chapter 1)
p216, line 17: in Section 2 of Chapter 1 $\longrightarrow$on page 77 (this is in Section 5 of Chapter 0)
p217, line 7: in Section 2 of Chapter 1 $\longrightarrow$on page 141 (this is in Section 1 of Chapter 1)
p220, line -4: in Section 2 of Chapter 1 $\longrightarrow$on page 147 (this is in Section 1 of Chapter 1)
p220, line -1: that section $\longrightarrow$pages 146,141
p227, line 14: $D{=}(g) \longrightarrow D{=}(f)$
p228, line 8: $\mathbb{C}^q \longrightarrow \mathbb{C}^g$
p228, line 16: $\Lambda_{2g} \longrightarrow \Pi_{2g}$

p229, line 16: $\int_{s_0}^{s} \longrightarrow \int_{p_0}^{s}$
p230, line 6: $\int_{s_0}^{s} \longrightarrow \int_{p_0}^{s}$
p235, line 7: $\varphi(D) \longrightarrow \mu(D)$
p235, 3rd display: left arrow should be pointing and is now an inclusion
p236, line -10: $\sum_i \longrightarrow \sum_\lambda$
p236, line -4: $(\mu^{(g)}(D')) \longrightarrow (\mu^{(g)}(D'))_j$
p236, line -1: $\mu^{(d)} \longrightarrow \mu^{(g)}$
p237, lines 2,4: $\mu^{(d)} \longrightarrow \mu^{(g)}$
p237, line -10: $df^*\omega \longrightarrow f^*\omega$
p238, line -5, RHS: $+[-2]$
p238, line -3: $\omega = dz \longrightarrow \omega = -2dz$
p239, line 14: $\omega = dz \longrightarrow dz$
p239, line 14: $\omega \longrightarrow \frac{1}{2}dz$
p239, lines -8,-1: $(\lambda) \longrightarrow (\Lambda)$
p239, line -4: Then $\longrightarrow$Since
p239, line -2, short sentence: under the assumption that RHS of previous display holds
p241, line 2: $s_0 \in S \longrightarrow s_0 \in S$
p241, lines 5,13,-5: $\int_{s_0} \longrightarrow \int_{p_0}$
p245, line -4: $h^0(K-D) > \max(0, g-d)$
p248, line 5: $h^0(K-D) \longrightarrow h^0(K-D)-1$; number $\longrightarrow$dimension of the space
p248, line 17: a $(d-r-1)$-plane $\overline{D}$
p251, Corollary: any *nondegenerate* curve
p251, Proof, line 2: second = should $\geq$ and the equality holds if and only if C is normal
p251, line -12: a *nondegenerate* curve
p252, line -7: $(l+m) \longrightarrow (l+m)-$
p252, line -3: in the section on rules surfaces $\longrightarrow$on page 533
p253, Noether's Theorem: $1 \longrightarrow l$

p472, line 4: extra)
p472, line 6: $\mathcal{O}(L') \longrightarrow \mathcal{O}_D(L')$
p474, line 18: $z_i \longrightarrow z(i)_i$
p474, line -1: $x \longrightarrow p$
p476, line -6: extra :
p477, line 7: $k+1 \longrightarrow k-1$
p477, line 10: $< \longrightarrow \leq$
p477, line 18: $m+1 \longrightarrow m+2$
p477, line -10: $L^1 \longrightarrow L'$
p478, lines -2,-1: $T_p(S) \longrightarrow T_p(S)$
p479, lines 1,3,4,5: $T_p(S) \longrightarrow T_p(S)$
p482, line -15: $\pi^{-1}(C)-E$ cannot contain $p_1 \in E$; this part should be just ignored
p484, line -3: extra *that*
p488, line 8: $\chi(\mathcal{O}) \longrightarrow \chi(\mathcal{O}_M)$

p491, lines -16,-15: from Section 2 of Chapter 3 $\longrightarrow$on page 396
p492, 4th displayed eqn from the bottom: *nondegenerate* rational maps
p493, line 1: *irreducible* analytic subvariety

p496, lines -9,-8: $(\mathbb{P}^{g-1})^*$; in Section 6 of Chapter 2 $\longrightarrow$on page 360 (this is in Sect. 7 of Chap. 2)
p498, line 2: $G_i\{a_j\}_{j\neq i}$ $\longrightarrow G_i$ in the notation on pp484,5
p500, bottom: the Poincare residue map is defined only for smooth C on p147 (in Sect. 1 of Chap. 1)
p508, line -13: $\big(\#f^{-1}(p_i)-1\big)$
p510, line -10: $\lambda\in\mathbb{P}^n$

p514, line -20: $\mathcal{O}S$ $\longrightarrow\mathcal{O}_S$ (twice); $\mathcal{O}C_\lambda$ $\longrightarrow\mathcal{O}_{C_\lambda}$
p515, like 2: first $\longrightarrow$ should be :
p516, line 8: missing $\longrightarrow$ before $E_x\otimes H_x^k$
p516, line 16: $H^0(\mathbb{P}^L,\mathcal{O}(E)$ $\longrightarrow H^0(\mathbb{P}^1,\mathcal{O}(E'))$; $h^0(E)$ $\longrightarrow h^0(E')-1$; of E $\longrightarrow$of E'
p516, line 17: $h^0(E)$ $\longrightarrow h^0(E')$
p517, line -5: $\circledast$ $\longrightarrow\otimes$
p521, line 7: at the end of the discussion on cubic surfaces $\longrightarrow$page 487
p521, lines -2,-1: this is the definition of $\pi(C_\lambda)$
p522, line -17: In Section 3 of Chapter 1 $\longrightarrow$On page 173
p522, line -6: once and away $\longrightarrow$once. Away
p525, Proposition, end of statement: or $\mathbb{P}^2\subset\mathbb{P}^2$
p525, line -11: $m_0\geq 3$
p525, line -10 and below: $m=m_0+1$
p525, line -8: on no line in S. Since
p527, line 12,13: Castelnuovo upper bound on page 252
p528, line 6: $n-1$ $\longrightarrow\mathbb{P}^{n-1}$
p528, line –4: $\big(L_1(\lambda)\cap L_2(\lambda)\big)$
p530, line 4: $\big(H_1(\lambda)\cap\ldots\cap H_n(\lambda)\big)$
p530, line 15: cut out by n quadrics
p533, line 6: $m(m-1)(n-1)/2+m\epsilon$
p533, lines -9,-8: of Section 3, Chapter 2 $\longrightarrow$on page 249
p534, line 11: $p_i\in H$ distinct
p534, line 16: at least 1 when non-empty.

p540, line 1: every line bundle
p540, line -7: ; $\longrightarrow$.
p556, line 7: $f=-\Psi^*\left(\dfrac{\partial g/\partial z_2}{\partial g/\partial z_1}\right)$
p556, lines -11,-8: C_λ $\longrightarrow C_{\lambda,i}$
p557, lines 11,12: are generically irreducible
p558, Lemma: any $\longrightarrow$some
p559, line 2: $K\cdot n_iD_i$ $\longrightarrow mK\cdot D_i$
p559, line 3: n_i^2 $\longrightarrow n_i$
p559, line -12: ψ^* $\longrightarrow\Psi^*$
p568, line 10: function around p
p574, line -1: n_i/m $\longrightarrow m/n_i$
p576, line -5: 0 should be appear on LHS
p579, line 14: $n-2$ $\longrightarrow n-1$