

MAT 566: Differential Topology

Problem Set 7

Here is a final collection of exercises, which will not be collected: 19-A, 19-B, ix, x, xi below.

Problem (ix): For this problem, assume Hurewicz Theorem (standard and torsion versions), homotopy l.e.s. for fibration, and that $\pi_i(S^{2n-1})$ is finite unless $i=2n-1$. Let

$$W^{2n-1} = \{(v, w) \in \mathbb{R}^{n+1} : |v|, |w| = 1, v \perp w\}.$$

- (a) If n is odd, to what simpler space is W^{2n-1} diffeomorphic to? What is its homology?
(b) Suppose n is even. Determine the cohomology and homology of W^{2n-1} , at least mod torsion. Determine $\pi_i(W^{2n-1})$ for $i \leq 2n-1$, at least mod torsion. Determine $\pi_i(S^n)$ for $i \leq 2n-1$, at least mod torsion.

Problem (x): Suppose $n \geq 2$, $\pi_i(X) = 0$ and $\pi_i(Y) = 0$ for all $i < n$, and the Hurewicz homomorphisms

$$h_i : \pi_i(X) \longrightarrow H_i(X; \mathbb{Z}) \quad \text{and} \quad \pi_i(Y) \longrightarrow H_i(Y; \mathbb{Z})$$

are isomorphisms mod torsion for all $i < 2n-1$. Show that

$$h_i : \pi_i(X \vee Y) \longrightarrow H_i(X \vee Y; \mathbb{Z})$$

is also an isomorphism mod torsion for all $i < 2n-1$.

Note: This is part of the proof of Theorem 18.3, but the book's proof is rather sketchy.

Problem (xi): Show that $\pi_n(S^1 \vee S^n)$ is not finitely generated for all $n \geq 2$.