# MAT 566: Characteristic Classes

## Course Information

### Course Instructor

*Name:* Aleksey Zinger | *Office:* Math Tower 3-111
*E-mail:* azinger@math | *OHs:* Tu 5-7 (or catch me some other time)

### Course Website

All updates, including schedule, homework assignments, and references, will be posted on the course website,

`http://math.stonybrook.edu/∼azinger/mat566-spr24`.

*Please visit this website regularly.*

### Prerequisites

This course is limited to the PhD students in mathematics who have passed their comps. All others must obtain permission from the instructor before registering for this course. Some familiarity with complex geometry (MAT 545) and algebraic topology (MAT 541) would be helpful. Feel free to contact me with any questions.

### Grading

Your grade will be based on class participation, class presentation, and light homework assignments *if you have not passed the orals yet* (roughly two problems every two weeks); see the next page for more details.

### Readings

The textbook is *Characteristic Classes*, Annals of Mathematics Studies 76, by John Milnor and James Stasheff. We will cover most of the book plus some of the readings listed on the next page (perhaps only the $S^7$-paper) and/or some other papers. You should acquire the book, but copies of the supplementary readings will be provided as needed.

### About the Course

Characteristic classes are certain cohomology classes associated to real and complex vector bundles. While they can be used to distinguish between non-isomorphic vector bundles, this is certainly not the main point of them. Characteristic classes of vector bundles often provide a convenient way for doing computations on manifolds which may lead to deep results in the topology of smooth manifolds. Most of the papers listed on the next page are essentially clever applications of these classes. Characteristic classes, as well as vector bundles, are indispensable in modern-day geometry and topology.

## Homework Assignments

Each section in the book is followed by a few exercises. These are generally directly related to the section and are thus not very hard. Two or so of these exercises will be assigned every two weeks or so. However, you should figure out all (or at least most) of the exercises for yourself. When writing solutions to the assigned exercises, you should take the statements of all preceding exercises as given. Feel free to discuss any of the exercises with anyone else, but do write your own solutions.

You should also read (and study in detail) every section of the book (as well as additional readings) covered in class. The book is relatively leisurely written, more like Spivak than Warner, and is a joy to read.

This is an intermediate grad course, and the formal requirements are fairly light. However, the more effort you put into this course, the more you are likely to benefit from it. If you are interested in algebraic geometry, at the minimum you should have a firm grasp of Chern classes and Grassmannians. If you are interested in algebraic topology, you should try to master the entire book, as many modern constructions build up on those in the book. If you are interested in geometry in general, this would be somewhere in between.

## Class Presentation

You will be asked to give a presentation during one of the classes on either a geometric (i.e. very concrete) application of characteristic classes or a related geometric topic. You will prepare the presentation with another student; the class time and the topic will be split between the two of you. You will need to meet with me before your presentation.

Here are some of the potential topics for in-class presentation, time-permitting:

- Schubert Calculus, based on Griffiths&Harris, pp197-207. *How many lines pass through* 4 *lines in* 3*-space?* [Cohomology of Grassmannians]
- *How many lines lie on a cubic surface? on a quintic threefold?*
- J. Milnor, *On manifolds homeomorphic to the* 7*-sphere*, Annals of Math. 64 (1956), 399–405 [existence of more than one differentiable structure on $S^7$]
- Hirzebruch's Signature Theorem
- M. Kervaire, *A manifold which does not admit any differentiable structure*, Comment. Math. Helv. 35 (1961), 1–14
- M. Atiyah and R. Bott, *The moment map and equivariant cohomology*, Topology 23 (1984), 1–28 [computation of characteristic classes by localization to fixed loci of group actions]
- J. Milnor, *Construction of universal bundles, II*, Ann. of Math. (2) 63 (1956), 430–436 [construction of classifying spaces]
- J. Milnor, *Topology from the Differentiable Viewpoint*, Princeton Landmarks in Mathematics [Sard's Theorem]