MAT 562: Symplectic Geometry

Course Instructor

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Course Website

All updates, including schedule, homework assignments, and references, will be posted on the course website,

`http://math.stonybrook.edu/∼azinger/mat562-fall26`.

Please visit this website regularly.

Prerequisites

This course is limited to the PhD students in mathematics who have passed their comps. All others must obtain permission from the instructor before registering for this course. Co-enrolling in MAT 545 (complex geometry) and MAT 541 (algebraic topology) would nicely complement this course. Feel free to contact the instructor with any questions.

Grading

Your grade will be based on class participation and light homework assignments *if you have not passed the orals yet* (roughly two problems every two weeks); see the next page for more details.

Readings

We will mostly follow *Foundations of Symplectic Geometry* by the instructor, prepared after teaching this course in Fall 2024. Many of the same topics are covered in *Introduction to Symplectic Topology*, 3rd ed., by Dusa McDuff and Dietmar Salamon, colloquially known as the *baby McDuff-Salamon* (as opposed to their mammoth J-holomorphic curves book) and *Lectures on Symplectic Geometry* by Ana Cannas da Silva. The latter is written in a rather light way, more like lecture notes than a book, as the title suggests, but could make for a more pleasant reading. The former goes far deeper into some introductory topics and includes more advanced topics, but does not cover some of the foundational topics properly. Furthermore, the authors use the negative of the standard definition of the Lie bracket of two vector fields on a smooth manifold (see footnote on p23 and the long remark on p98), the notation $\mathrm{d}f(x)v$ for the differential of smooth map f at a point x applied to the tangent vector v (instead of $\mathrm{d}_x f(x)$), and no commas (which oftentimes would have helped with the readability).

About the Course

The mathematical field of symplectic geometry and topology arose in the 1950s as a formalization of the equations of motions of classical physics into the study of Hamiltonian flows on symplectic manifolds, now the subfield of SG/ST called Hamiltonian dynamics. Since then, this field has greatly expanded to include such subfields as the geography of symplectic manifolds, i.e. what manifolds could potentially be made symplectic, and the theory of pseudoholomorphic curves, which underpins the connections of SG/ST to string theory and algebraic geometry. This course will focus on the very foundations of SG/ST that are generally relevant throughout this field. These include the local structure of symplectic manifolds and submanifolds (Darboux Theorem, equivariant version), almost complex structures, some restrictions on the topology of symplectic manifolds, various constructions of symplectic manifolds (symplectic reduction, cut, and sum), Atiyah-Guillemin-Sternberg Convexity Theorem (description of images of moment maps), and Delzant Theorem (classification of symplectic manifolds with "maximal" torus actions).

The *Introduction* and *Chapter 1* of McDuff-Salamon motivate the field of SG/ST and translate the equations of motions of classical physics into the mathematical language of this field; you should read them on your own. *Chapter 2* covers background linear algebra and establishes linear specializations of some of the remarkable SG/ST results established in greater generality later in the book and/or elsewhere in the literature. Please read these on your own before the first lecture, as they are not part of the instructor's manuscript and will not be covered in lecture. We will cover analogues of the foundational *Chapters 3* and *4* of McDuff-Salamon thoroughly, followed by analogues of parts of the more specialized *Chapters 5-7* and more, possibly including various symplectic cut constructions and the Delzant Theorem on toric symplectic manifolds.

Of the intermediate graduate courses in geometry offered in the Fall, this course will arguably have the most topological flavor, in large part due to the proof of Atiyah-Guillemin-Sternberg Convexity Theorem. On the other hand, it would have far more geometric content than any of the core or intermediate graduate courses in topology offered in the Fall.

Homework Assignments

There are exercises dispersed throughout the instructor's manuscript and the two aforementioned textbooks. These are generally directly related to the text and are thus not very hard. Two or so of these and related exercises will be assigned every two weeks or so. However, you should figure out all (or at least most) of the exercises for yourself. When writing solutions to the assigned exercises, you should take the statements of all preceding exercises as given. Feel free to discuss any of the exercises with anyone else, but do write your own solutions.

You should also read (and study in detail) every section of the manuscript (as well as additional readings) covered in class. The manuscript is thoroughly written, so this should not be too hard.

This is an intermediate grad course, and the formal requirements are fairly light. However, the more effort you put into this course, the more you are likely to benefit from it. Please try to discuss this course with each other and come with any additional questions to office hours.