MAT 545: Complex Geometry
Fall 2008

Course Information

Course Instructor

Name: Aleksey Zinger *Office:* Math Tower 3-111 *Phone:* 632-8288
E-mail: azinger@math.sunysb.edu *Office Hours:* Tu 5-7 in 3-111, W 9-10 in P-143

Course Website

All homework assignments and various updates will be posted on the course website,

`http://math.sunysb.edu/~azinger/mat545`.

Please visit this website regularly.

Prerequisites

This is an intermediate-level graduate course, continuing the core courses (primarily 531 and 544 content-wise, but the other core courses as well method-wise). You should thus pass the comps before taking this course. If you would like to take it without first passing the math department comps, please contact me. If you are taking the math comps in August, please feel free to register for the course beforehand.

Grading

Your grade will be based on class participation and light homework assignments (two to three problems every two weeks).

Textbook

The textbook is *Principles of Algebraic Geometry* by Phillip Griffiths and Joseph Harris. We will cover Chapters 0 and 1 (at least most of them). The book is pricey for a paperback, but its 800+ pages pack a wealth of beautifully presented information on fundamentals of algebraic geometry from a geometric (and analytic) viewpoint. While the textbook is required for the course, the bookstore may not stock it; please acquire it ahead of time elsewhere (e.g. at amazon which usually sells it at 20% off the list price with free shipping).

Course Description

Complex manifolds play a prominent role throughout geometry. In addition to being fundamental to algebraic geometry, many areas of differential geometry, and string theory, they provide a wealth of concrete example in such fields as symplectic topology and topology of 4-manifolds. The most common modern approach to complex manifolds is heavily algebraic, such as in Robin Hartshorne's *Algebraic Geometry*. In contrast, the approach taken in *Principles of Algebraic Geometry* is geometric (and thus analytic), though some amount of algebra is unavoidable (cohomology arises from an algebraic construction after all).

Much of Chapter 0 is an efficient overview of the material covered (or that could have been covered) in core courses, but with a very different twist. The focus here is on the tools used in complex geometry: complex analysis, Stokes' theorem, global analysis, and algebraic topology. These tools are used in Chapter 1 to establish the foundations of complex and algebraic geometry such as embedding theorems for complex manifolds and vanishing theorems for certain cohomologies. In later chapters, they are used to study complex curves (Riemann surfaces) and complex surfaces (complex 2-manifolds) in depth.

There are close relations and even overlaps between MAT545, MAT536 (*Algebraic Geometry*), and MAT543 (*Complex Analysis II*). MAT536 introduces similar material, but from a purely algebraic perspective; MAT543 focuses more on the analytic aspects of this course. These two courses are in many ways complementary to MAT545, and taking one of them in addition to MAT545 might make a lot of sense depending on your interests.

Homework Assignments

If you want to learn this material, it is essential to work thoroughly through the text and do as many exercises as possible. While the book itself contains no exercises, I will distribute problem sets every two weeks or so. Each problem will be worth 5 or 10 points, and you will be expected to write up solutions to a subset of problems worth 20 points from each problem set and hand them in to me by the due date; *late problem sets will not be accepted.* In addition to the problems you write up, you should try to figure out all (or at least most) of the remaining exercises for yourself.

Feel free to discuss any of the exercises with anyone else, but do write your own solutions. You are also encouraged to come to the office hours.