MAT 542: Algebraic Topology, Fall 2016

Suggested Problems for Week 4

You may hand in solutions to at *most* 2 problems every 2 weeks and no later than 2 weeks after the necessary material for them is covered in class.

From Munkres: 12.4, 12.5, 13.6, 11.1

Problem G

Suppose R is a commutative ring with 1 and $(\mathcal{C}_*, \partial)$ and $(\mathcal{C}'_*, \partial')$ are chain complexes over R such that $(\mathcal{C}_*, \partial)$ is free, $\mathcal{C}_p = 0$ for $p < 0$, and $H_p(\mathcal{C}', \partial') = 0$ for all $p > 0$. Let

$$g\colon H_0(\mathcal{C}_*, \partial) \longrightarrow H_0(\mathcal{C}'_*, \partial')$$

be a homomorphism of R-modules. Show that there exists a chain map

$$f_*\colon (\mathcal{C}_*, \partial) \longrightarrow (\mathcal{C}'_*, \partial') \qquad \text{s.t.} \quad f_{0*} = g\,.$$

This problem is from the midterm in MIT's 18.905 in Fall 1996 taught by F. Peterson.

Problem H

(a) State and prove excision for relative ordered simplicial homology. Show that the excision isomorphisms in the ordered and oriented homologies commute with the canonical isomorphisms between the two homologies.

(b) State and prove Mayer-Vietoris for ordered simplicial homology. Show that the Mayer-Vietoris long exact sequences in the ordered and oriented homologies commute with the canonical isomorphisms between the two homologies.