MAT 541: Algebraic Topology

Suggested Problems for Week 9

You may hand in solutions to at *most* 2 problems every 2 weeks and no later than 2 weeks after the necessary material for them is covered in class.

From Munkres: 37.1-3, 38.2-4, 39.2, 36.2, 36.3

Problem O

Let $n\in\mathbb{Z}^+$ and $S^{2n-1}\subset\mathbb{C}^n$ be the unit sphere. For $p\in\mathbb{Z}^+$ and $q_1,\ldots,q_n\in\mathbb{Z}$ such that each q_i is prime relative to p, define

$$L\big(p;q_1,\ldots,q_n\big)=S^{2n-1}/\!\sim,$$
$$\big(z_1,\ldots,z_n\big)\sim\big(\mathrm{e}^{2\pi\mathrm{i}\,kq_1/p}z_1,\ldots,\mathrm{e}^{2\pi\mathrm{i}\,kq_n/p}z_n\big)\quad\forall\ \big(z_1,\ldots,z_n\big)\in S^{2n-1},\ k\in\mathbb{Z}.$$

(a) Show that $L(p;q_1,\ldots,q_n)$ is a compact connected smooth manifold of dimension $2n$–1.

(b) Show that

$$L(1;1,\ldots,1)=S^{2n-1},\qquad L(2;1,\ldots,1)=\mathbb{RP}^{2n-1},$$
$$L\big(p;q_1,\ldots,q_i,\ldots,q_n\big)=L\big(p;q_1,\ldots,-q_i,\ldots,q_n\big),$$
$$L\big(p;q_1,\ldots,q_i,q_{i+1},\ldots,q_n\big)=L\big(p;q_1,\ldots,q_{i+1},q_i,\ldots,q_n\big),$$
$$L\big(p;q_1,\ldots,q_n\big)=L\big(p;qq_1,\ldots,qq_n\big)$$

for any $q\in\mathbb{Z}$ relatively prime to p.

(c) Show that $L(p;1,q)$ is homeomorphic to the space $L(p;q)$ of Section 40 in Munkres and obtain the claim of 40.4 in Munkres without any triangulations.

(d) Compute π_1 and H_* of $L(p;q_1,\ldots,q_n)$.