MAT 531 Geometry/Topology Midterm

1. (20 %) Is the parameterized curve

$$x = t^2, \quad y = t^3$$

a smooth submanifold of $\mathbb{R}^2$? As a topological space, does it have a smooth structure? Explain your answer, but you may skip details.

2. (20 %) Compute the integral

$$\int_{x^2+y^2+z^2=1} x\, dy \wedge dz.$$

3. (20 %) Give an example of two different (non-compatible) smooth atlases on $\mathbb{R}$.

4. (20 %) Prove that the vector field $3z^2\partial_x + 2x\partial_z$ is tangent to the surface $x^2 + y^2 - z^3 = 0$ at all points where this surface is smooth.

5. (20 %) Consider vector fields $v = \partial_x$ and $w = \partial_y$ on the plane $z = 1$ in $\mathbb{R}^3$. Let f be the radial projection of this plane to the sphere $x^2 + y^2 + z^2 = 1$. Compute the commutator of $f_*(v)$ and $f_*(w)$.

6. (20 %) Compute the curl of the vector field $x\partial_y + y\partial_z + z\partial_x$ on $\mathbb{R}^3$.

7*. (25 %) Let v and w be vector fields on a manifold X tangent to a submanifold $Y \subset X$. Prove that $[v, w]$ is also tangent to Y.