MAT 531: Topology&Geometry, II
Spring 2006

Problem Set 3
Due on Thursday, 2/16, in class

1. (a) For what values of $t \in \mathbb{R}$, is the subspace

$$\left\{(x_1,\ldots,x_{n+1}) \in \mathbb{R}^n : x_1^2+\ldots+x_n^2-x_{n+1}^2 = t\right\}$$

a smooth embedded submanifold of $\mathbb{R}^{n+1}$?
(b) For such values of t, determine the diffeomorphism type of this submanifold (i.e. show that it is diffeomorphic to something rather standard).
Hint: Draw some pictures.

2. Show that the special unitary group

$$SU_n = \left\{A \in \mathrm{Mat}_n\mathbb{C} : \bar{A}^t A = \mathrm{I}_n,\ \det A = 1\right\}$$

is a smooth compact manifold. What is its dimension?

3. Suppose that $f : M \longrightarrow N$ is a smooth map and $\pi : V \longrightarrow N$ is a smooth vector bundle. The pullback of V by f, $\pi_1 : f^*V \longrightarrow M$, is the vector bundle defined by taking

$$f^*V = \left\{(m,v) \in M \times V : f(m) = \pi(v)\right\} \subset M \times V.$$

In particular, f^*V is supposed to be a smooth manifold. Use the Implicit Function Theorem to show that f^*V is in fact a smooth submanifold of $M \times V$.

4. Chapter 1, #22 (p51)

5. Chapter 1, #17 (p51)

6. Let V be the vector field on $\mathbb{R}^3$ given by

$$V(x,y,z) = y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y} + \frac{\partial}{\partial z}.$$

Explicitly describe and sketch the flow of V.