MAT 531: Topology&Geometry, II
Spring 2006

Problem Set 1
Due on Thursday, 2/2, in class

Give concise, but complete, solutions. The entire problem set should not require more than a few pages.

Please read Notes on Vector Bundles, posted on the website, before starting on Questions 4-6.

1. Chapter 1, #2 (p50)

2. Verify that the differential $d\psi$ of a smooth map $\psi: M \longrightarrow N$, as defined in 1.22 (p16), is indeed well-defined. In other words, $d\psi(v)$ is a derivation on $\tilde{F}_{\psi(m)}$ for all $v \in T_m M$ and $m \in M$.

3. Chapter 1, #5 (p50)

4. (a) Show that the quotient topologies on $\mathbb{C}P^n$ given by $(\mathbb{C}^{n+1}-0)/\mathbb{C}^*$ and S^{2n+1}/S^1 are the same (i.e. the map $S^{2n+1}/S^1 \longrightarrow (\mathbb{C}^{n+1}-0)/\mathbb{C}^*$ induced by inclusions is a homeomorphism).
(b) Show that $\mathbb{C}P^n$ is a compact topological $2n$-manifold. Furthermore, it admits a structure of a *complex* (in fact, *algebraic*) n-manifold, i.e. it can be covered by charts whose overlap maps, $\varphi_\alpha \circ \varphi_\beta^{-1}$, are holomorphic maps between open subsets of $\mathbb{C}^n$ (and rational functions on $\mathbb{C}^n$).
Note: you can do this with $n+1$ charts.
(c) Show that $\mathbb{C}P^n$ contains $\mathbb{C}^n$, with its complex structure, as a dense open subset.
(d) Show that the tautological line bundle $\gamma_n \longrightarrow \mathbb{C}P^n$ is indeed a complex line bundle (describe its trivializations). What is its transition data?

5. Show that the tangent bundle TM of a smooth n-manifold is a real vector bundle of rank n over M. What is its transition data?

6. Show that the tangent bundle TS^1 of S^1, defined as in 1.25 (p19), is isomorphic to the trivial real line bundle over S^1.
Hint: Use a lemma from *Notes on Vector Bundles*.