Topology MAT 530 Homework 1. **1.** Show that, for any collection of topologies on a set, there is the smallest (coarsest, weakest) topology containing them all and the largest (finest, strongest) topology contained in them all.

2. Find a countable basis in $\mathbb{R}^2$ (with the standard topology given by a Euclidean metric).

3. A map $f : X \to Y$ is said to be an *open map* if for every open subset U of X, the set $f(U)$ is open in Y. Show that the natural projections $X \times Y \to X$ and $X \times Y \to Y$ are open maps.

4. Prove that

$$\overline{\bigcup A_\alpha} \supset \bigcup \overline{A_\alpha}$$

for an arbitrary collection of sets A_α in a topological space. Give an example where equality fails.

5. Prove that any open interval is homeomorphic to the real line.

6*. Give an example of a function $F : \mathbb{R}^2 \to \mathbb{R}$ such that for any $x \in \mathbb{R}$ the functions $y \mapsto F(x, y)$ and $y \mapsto F(y, x)$ are continuos, but the function F is not continuous.