MAT 319/320: Basics of Analysis, Spring 2019 Homework Assignment 2

Please read Sections 7-9 of Ross's textbook thoroughly.

Optional supplemental reading for MAT 320: Rudin's book, pp47-51

Problem Set 2 (due before the start of recitation on Wednesday, February 13th): 7.1d, 7.2, 7.4, 8.1d, 8.5, 8.8b, 9.1b, 9.3, 9.9, 9.11a, and Problem A* below * Problem A must be answered on a printout of this sheet

Problem A

Here are several "mixed-up" versions of the condition for convergence of a sequence $(s_n)_{n\in\mathbb{N}}$.

- (1) There exists a real number s such that for every $\epsilon > 0$ and every $n \in \mathbb{N}$, $|s_n s| < \epsilon$.
- (2) There exist real numbers s and $\epsilon > 0$ such that for all $n \in \mathbb{N}$, $|s_n s| < \epsilon$.
- (3) There exist a real number s and integer N > 0 such that for all $\epsilon > 0$ and n > N, $|s_n s| < \epsilon$.
- (4) There exists a real number s such that for every $\epsilon > 0$, there exists N > 0 such that for n > N, $|s_n s| < 100\epsilon$.
- (5) For every real number s, there exists $\epsilon > 0$ such that for all $n \in \mathbb{N}$, $|s_n s| < \epsilon$.
- (6) For every real number s, there exists $\epsilon > 0$ and $n \in \mathbb{N}$ such that $|s_n s| < \epsilon$.

| (7) For every real number s and for every $\epsilon > 0$, there exists $n \in \mathbb{N}$ such that $ s_n - s < \epsilon$. (8) For every real number s and every $\epsilon > 0$, there exists $N > 0$ such that for $n > N$, $ s_n - s < \epsilon$. |
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| Which conditions above are equivalent to boundedness? () () |
| Which condition above is equivalent to convergence? () |
| Which condition above is satisfied by $every$ sequence of real numbers? () |
| Which condition above is satisfied by no sequence of real numbers? () |
| For each of the three remaining conditions, give a simpler description in your own words of what the condition tells you about the sequence s_n . |
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