MAT 531 SPRING 16 HOMEWORK 12

Due Tuesday, May 3

1. Let ω be the $(n-1)$-form on $\mathbb{R}^n \setminus \{0\}$ defined by

$$\omega = |x|^{-n} \sum_{i=1}^{n} (-1)^{i-1} x^i dx^1 \wedge \cdots \wedge \overset{\wedge}{dx^i} \wedge \cdots \wedge dx^n.$$

(a). Compute

$$\int_{S^{n-1}(1)} \omega.$$

directly - without using Stokes theorem. Here $S^{n-1}(1)$ is the unit sphere in $\mathbb{R}^n$, with orientation induced by the outward unit normal.

(b). Compute the integral using Stokes theorem.

(c). Show that ω is closed, but not exact on $\mathbb{R}^n \setminus \{0\}$.

2. Let ω be a closed non-degenerate 2-form on a compact manifold M. (ω is called a symplectic structure on M). Thus, the rank of ω is n - see earlier HW - Spivak, Ch. 7, Prob. 8

(a). Show that $\omega^n = \omega \wedge \cdots \wedge \omega$ (the n-fold wedge product) is a non-vanishing $2n$ form on M.

(a). Show that $\omega^n = \omega \wedge \cdots \wedge \omega$ (the n-fold wedge product) is not exact.

(b) Show that $H^{2k}_{deR}(M) \neq 0$ for all $1 \leq k \leq n$.

3. Prove that $\mathbb{RP}^n$ is orientable if and only if n is odd. Hint: use the characterization of orientability as existence of a non-vanishing n-form and work mostly on the universal cover S^n which is orientable.

4. Let $F : M \to N$ be a smooth map of compact, orientable n-manifolds. Prove that if F is not surjective, then

$$degF = 0.$$