PRACTICE MIDTERM FOR MAT 341

(1) Consider the function $f(x) = 2x - 1, \quad 0 < x < 2$.

(a) Compute the Fourier sine series for $f(x)$. At each $x \in [-4, 6]$ compute the value of this series.

(b) Compute the Fourier cosine series for $f(x)$. At each $x \in [-4, 6]$ compute the value of this series.

(2) Set $f(x) = 2 + \sum_{n=1}^{\infty} \frac{cos(nx)}{n^4} + \sum_{n=1}^{\infty} \frac{sin(nx)}{n^2}$.

(a) Explain why $f(x)$ converges for each value of x.

(b) Explain why $f(x)$ is a periodic function of period 2π.

(c) Explain why $f(x)$ is a continuous function.

(d) For each positive integer n find the value of the integral $\int_{-\pi}^{\pi} f(x)cos(nx)dx$.

(3) Set $w(x,t) = sin(rx)e^{st}$.

(a) Show that $w(x,t)$ statisfies

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{k}\frac{\partial w}{\partial t}$$

iff $-r^2 = \frac{s}{k}$.

(b) Show that $w(x,t)$ satisfies all of the following equalities

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{k}\frac{\partial w}{\partial t}$$

$$w(0,t) = 0, w(a,t) = 0$$

iff $r = \frac{n\pi}{a}$ and $s = -\frac{kn^2\pi^2}{a^2}$.

(4) Find $u(x,t)$ which satisfies all the following equalities:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\frac{\partial u}{\partial t}$$

$$u(0,t) = 0, \quad u(\pi,t) = 1$$

$$u(x,0) = \frac{x}{\pi} + 3sin(2x) - sin(7x).$$

What is a physical situation which these equations describe?

(5) Consider the equations

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

$$u(0,t) = T_0, \quad -\kappa\frac{\partial u}{\partial x}(a,t) = (u(a,t) - T_1)h$$

where κ, h are positive constants and T_0, T_1 are constants.

(a) What is a physical situation that these equations describe?
(b) Find the "steady state" solution to these equations.
(c) Why is the "steady state" solution important – both physically and mathematically?