MAT 341 HW 2

1.5

(2) $f(x) = \frac{\pi - x}{2}$ $0 < x < 2\pi$

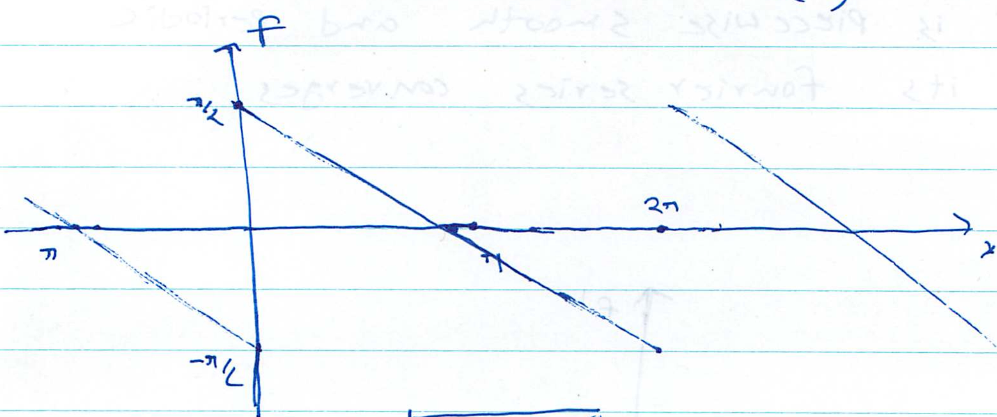
Its Derivative

$f'(x) = -1/2$

and $F(x) = \int_0^x f(t)\,dt$

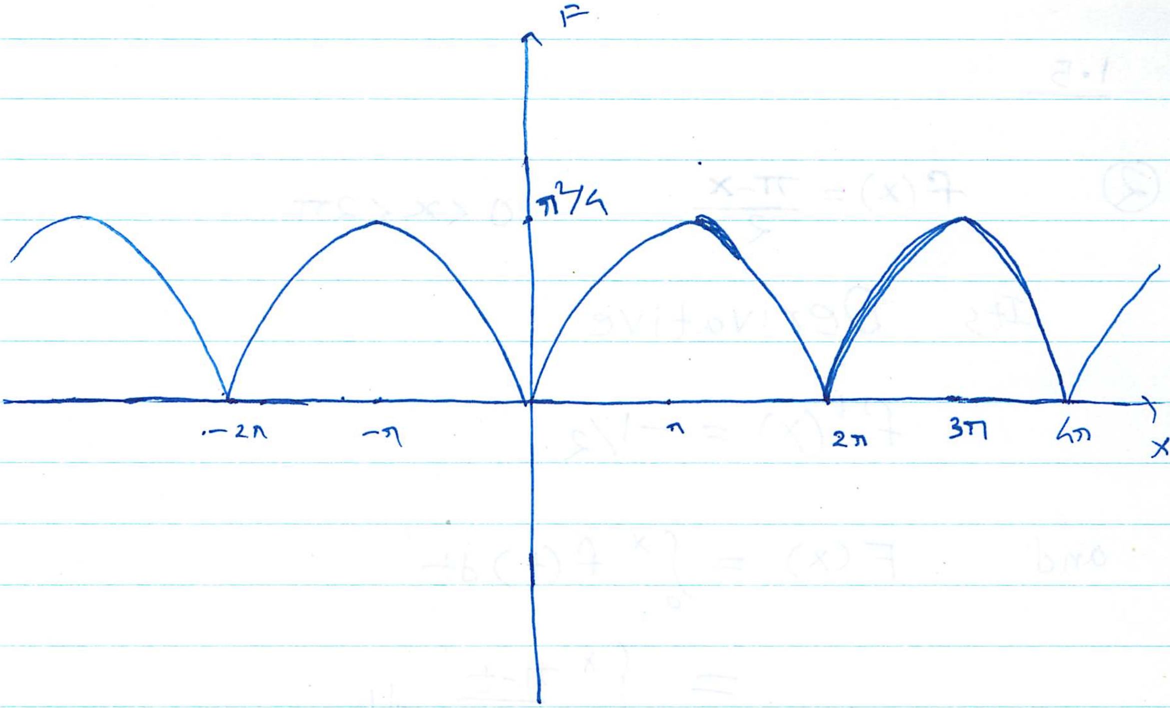
$= \int_0^x \frac{\pi - t}{2}\,dt$

$= \frac{1}{2}\left(\pi x - \frac{x^2}{2}\right)$



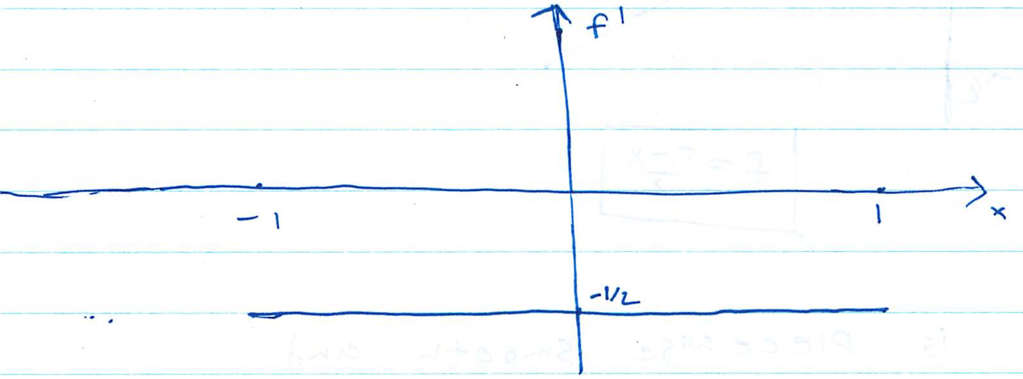
$\boxed{f = \frac{\pi - x}{2}}$

f is piecewise smooth and Periodic so at each point fourier series converges. (Though at the point of discontinuities it takes value 0 which may be different from f depending on its defn)



$$F(x) = \frac{1}{2}\left(\pi x - x^2/2\right)$$

F is piecewise smooth and periodic so its fourier series converges.



$$f(x) = -\frac{1}{2}$$

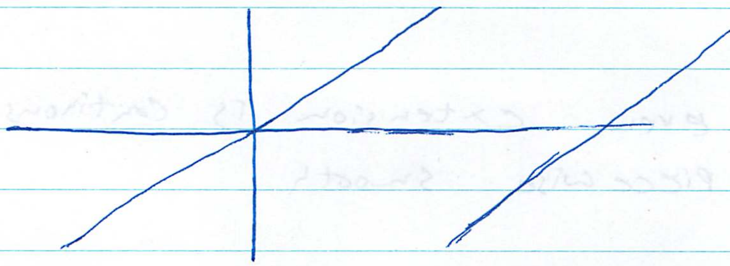
f' is piecewise smooth and periodic, so it also has convergent fourier series.

$\boxed{3}$

$$f(x) = x \qquad 0 < x < \pi$$

It's derivative,

$$f'(x) = 1 \qquad 0 < x < \pi$$



The odd extension of f is not continous

has $f'(x) = 1$ every where

its given by

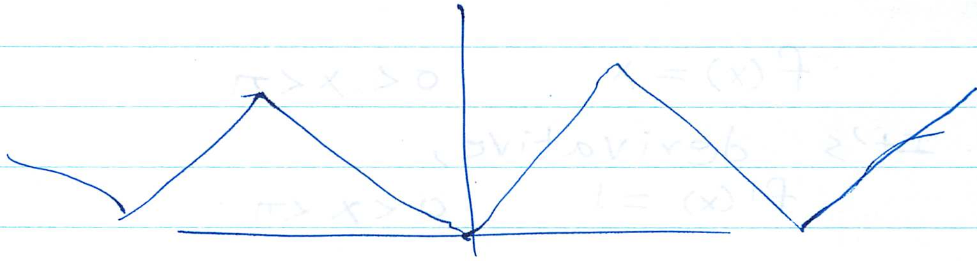
$$f(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$

So the Fourier Sine series

can't be differentiated term by term

as $2 \sum_{n=1}^{\infty} (-1)^{n+1} \cos(nx)$ doesn't converge

for example at $x=0$



The even extension is continous and piecewise smooth

The cosine series is given by

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \cdots\right)$$

The differentiated series is

$$\frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \cdots\right)$$

It always converges to f' except a those points where f' are not differentiable i.e. $\pm 2n\pi$. (see Example Page 81)

(8)

$$P(x) = x(\pi - x) \qquad 0<x<\pi$$

$$\left(x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx) \qquad 0<x<\pi \right)$$

Since x is differtiable so in Particular its Periodic extension is Piecewise continous

So We may integrate the series (0 to x) term by term to get

$$\frac{x^2}{2} = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{-\cos(nx)}{n} \right) + C \quad 0<x<\pi$$

$$\Rightarrow x^2 = 4\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n^2} \cos(nx) + 2C$$

$$\therefore \text{Where } C = -2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{-\cos(n\cdot 0)}{n} \right)$$

$$= 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$a_0 = -2C = 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\therefore P(x) = x(\pi - x) = \pi x - x^2$$

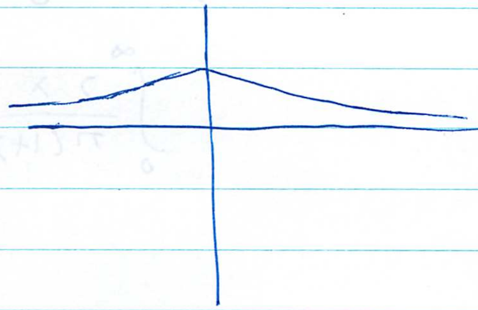
$$\therefore P(x) = a_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (2\pi) \sin(nx)$$

$$- 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(nx)$$

$\boxed{1.9}$

1a. $f(x) = e^{-x}$ $\quad 0 < x < \infty$

For the even extension we have cosine Representation



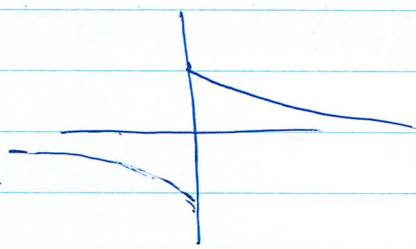
$$A(\lambda) = \frac{2}{\pi} \int_0^{\infty} e^{-x} \cos \lambda x \; dx$$

$$= \frac{2}{\pi} \left| \frac{e^{-x} (\lambda \sin(\lambda x) - \cos(\lambda x))}{1+\lambda^2} \right|_0^{\infty}$$

$$= \frac{2}{\pi(1+\lambda^2)}$$

$$f(x) = \int_0^{\infty} \frac{2}{\pi(1+\lambda^2)} \cos \lambda x \; d\lambda$$

For odd extension



$$B(\lambda) = \frac{2}{\pi} \int_0^{\infty} e^{-x} \sin(\lambda x) \, dx$$

$$= \frac{2}{\pi} \left| \frac{e^{-x}(-\lambda \cos(\lambda x) - \sin \lambda x)}{1+\lambda^2} \right|_0^{\infty}$$

$$= \frac{2\lambda}{\pi(1+\lambda^2)}$$

f (x) is given by

$$\int_0^\infty \frac{2\lambda}{\pi(1+\lambda^2)} \sin(\lambda x)\, d\lambda$$

3a

$f(x) = \frac{1}{1+x^2}$, We have

$$A(\lambda) = \frac{2}{\pi} \int_0^\infty \frac{1}{1+x^2} \cos \lambda x \, dx$$

$$= e^{-|\lambda|}$$ (This is done in the book See Example 3 Page 102)

$$B(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1+x^2} \sin \lambda x \, dx$$
$$= 0$$

The integral representation is

$$\therefore \quad f(x) = \int_0^\infty e^{-\lambda} \cos \lambda x \, d\lambda$$

3b

$f(x) = \frac{\sin x}{x}$, We get

$$A(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin x}{x} \cos \lambda x \, dx$$

$$= h(\lambda)$$

where $h(\lambda) = \begin{cases} 1, & \text{if } |\lambda| < 1 \\ 0, & \text{if } |\lambda| > 1 \end{cases}$

(see Example 2 Page 101)

$$B(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} \underbrace{\frac{\cos x}{x} \cos \lambda x}_{\text{odd}} \, dx = 0$$

So The integral representation of $f(x)$ is given by

$$f(x) = \int_0^1 \cos(\lambda x)\, d\lambda$$

2.1

(2)

(i) Let

$u(x,t) = \exp(-\lambda^2 \kappa t)\cos(\lambda x)$

$$\frac{\partial u}{\partial x} = -\lambda \exp(-\lambda^2 \kappa t)\sin(\lambda x)$$

$$\frac{\partial^2 u}{\partial x^2} = -\lambda^2 \exp(-\lambda^2 \kappa t)\cos(\lambda x)$$

$$\frac{\partial u}{\partial t} = -\lambda^2 \kappa \exp(-\lambda^2 \kappa t)\cos(\lambda x)$$

So $\rightarrow$ $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\kappa}\frac{\partial u}{\partial t}$$

(ii) Let

$u(x,t) = \exp(-\lambda^2 \kappa t)\sin(\lambda x)$

$$\frac{\partial u}{\partial x} = \lambda \exp(-\lambda^2 \kappa t)\cos(\lambda x)$$

$$\frac{\partial^2 u}{\partial x^2} = -\lambda^2 \exp(-\lambda^2 \kappa t)\sin(\lambda x)$$

$$\frac{\partial u}{\partial t} = -\lambda^2 \kappa \exp(-\lambda^2 \kappa t)\sin(\lambda x)$$

So again

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\kappa}\frac{\partial u}{\partial t}$$