MAT 310 FALL 09 HOMEWORK 7

Due Wednesday, November 4

1. Let $v = (1, 2+i, i, 1-2i)$ and $w = (2-i, 1, 3+2i, 3i)$ be vectors in $\mathbb{C}^4$. With respect to the standard (Euclidean) inner product on $\mathbb{C}^4$, compute $\langle v, w\rangle$, $|v|$, $|w|$ and $|v+w|$. Then verify both the Cauchy-Schwarz and triangle inequalities for these vectors.

2. Let $V = C^0([0,1])$ be the vector space of continous, real-valued functions on the interval $[0,1]$, and define

$$\langle f, g\rangle = \int_0^{1/2} f(t)\cdot g(t)dt.$$

Is this an inner product on V? Prove or disprove.

3. Let $T : V \to V$ be a linear map on an inner product space $(V, \langle\cdot,\cdot\rangle)$. Suppose that

$$|T(x)| = |x|,$$

for all $x \in V$. Prove that T is 1-1 and onto, so that T is an isomorphism.

4. Let B be a basis for a finite dimensional inner product space.
   (a). Prove that if $\langle x, v_i\rangle = 0$ for all $v_i \in B$, then $x = 0$.
   (b). Prove that if $\langle x, v_i\rangle = \langle y, v_i\rangle$, for all $v_i \in B$, then $x = y$.

5. Let $P_2([-1,1])$ be the vector space of real-valued polynomials on $[-1,1]$ of degree ≤ 2, and given the L^2 inner product as in Problem 2. Let $(1, x, x^2)$ be the standard basis of $P_2([-1,1])$. Find an orthonormal basis (v_1, v_2, v_3) associated to $(1, x, x^2)$ (by the Gram-Schmidt process).

6. Let S be the subspace spanned by the vectors $(1, 0, i)$ and $(1, 2, 1)$ in $\mathbb{C}^3$. Compute $S^\perp$ in $\mathbb{C}^3$, (standard inner product).

7. If U is a subspace of a finite dimensional vector space V, find a formula for the $dim\, U^\perp$ in terms of $dim\, U$ and $dim\, V$, and prove your formula is correct.