MAT 310 FALL 09 HOMEWORK 6

Due Wednesday, October 28

1. Find all eigenvalues and eigenvectors of the linear map

$$T : \mathbb{R}^3 \to \mathbb{R}^3,$$
$$T(x_1, x_2, x_3) = (x_2, x_3, x_1).$$

2. Suppose $T : V \to V$ is a linear operator on V, with V a finite dimensional vector space over $\mathbb{F}$. Prove that every vector in the null space of T is an eigenvector, with eigenvalue 0 and conversely, every such eigenvector is in the nullspace of T. Conclude that $null(T)$ equals the span of the 0-eigenvectors.

3. Suppose $T : V \to V$ is a linear operator on V, with V a finite dimensional vector space over $\mathbb{F}$. If $dim(range\, T) = 1$, show that T can have at most 2 distinct eigenvalues. (Hint: Use (2)).

4. Suppose λ is an eigenvalue of $T : V \to V$. Prove then that λ^2 is an eigenvalue of $T^2 = T \circ T : V \to V$, and similarly, λ^k is an eigenvalue of $T^k : V \to V$.

Is the converse true? So if μ is an eigenvalue of T^2, is $\sqrt{\mu}$ an eigenvalue of T? If yes, prove it, if no, give an example.

5. Consider the 4×4 matrix

$$A = \begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & -1 & 8 & 5 \\ 0 & 0 & 6 & -4 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

This gives a linear map $A : \mathbb{R}^4 \to \mathbb{R}^4$. Show that the standard "horizontal" subspaces $\mathbb{R}^k = \{x_{k+1} = \cdots = x_4 = 0\}$, $k = 1, 2, 3$ are invariant subspaces of A.