MAT 310 FALL 09 HOMEWORK 10

Due Wednesday, December 2

1. Let $T : V \to V$ be a nilpotent linear map, so $T^k = 0$ for some $k < \infty$. Prove that if v is a vector such that

$$T(v) = \lambda v,$$

then $\lambda = 0$, i.e. the only eigenvalue of a nilpotent operator is 0.

2. Prove that if $T : V \to V$ is an injective linear map then T cannot be nilpotent.

3. (a) Find all eigenvalues of the linear map $T : \mathbb{C}^3 \to C^3$ given by $T(z_1, z_2, z_3) = (z_2, 0, 3z_3)$. Determine the multiplicities of these eigenvalues.

(b). Find all generalized eigenvectors of T, corresponding to the eigenvalues found in (a).