Math 303: Fall 2007
Midterm 1: Review Sheet

No calculators, notes, or books will be allowed on the mid-term. The exam will consist of around 7 questions and last 55 minutes. It is essential that you clearly and neatly show all work in order to receive full or partial credit on the problems. Remember that your goal should not just be to arrive at the correct answer; you should convince the grader that you arrived at your answer by a correct (and followable) method.

For Midterm 1, you may be asked to:

- Write a differential equation which models a situation in the natural world.
- Solve a first-order differential equation. This includes separable, linear, homogeneous, and exact equations. You should know how to find a general solution and solve an initial value problem.
- Reduce certain second-order equations to first-order equations.
- Find equilibrium solutions and their stability. You should understand their relationship to a physical situation which the equation models.
- Determine the existence and uniqueness of solutions for a first-order equation.

The best way to study is by working problems, both old and new. Please review your old homeworks, including any comments, and work out new problems as well. Here are a few sample problems, with similar problems from the book referenced.

1. (§1.1: 1-12) Show that the function y satisfies the differential equation

$$y' + 2y = 0; y = 3e^{-2x}.$$

2. (§1.1: 27-36, §2.1, §2.3) Write a differential equation that describes the following situation: The acceleration dv/dt of Lamborghini is proportional to the difference between 250 km/h and the velocity of the car. Also, determine any equilibrium solutions and their stability, as well as what they mean in this physical situation. Solve the differential equation.

3. (§1.4: 1-18) Solve the initial value problem:

$$\frac{dy}{dx} = 3x^2(y^2 + 1); \quad y(0) = 1.$$

4. (§1.5: 1-25) Solve the equation

$$2xy' + y = 10\sqrt{x}.$$

5. (§1.6: 1-15) Solve the equation

$$x^2y' = xy + y^2.$$

6. (§1.6: 16-18, 26-30) Solve the equation

$$xe^y y' = 2(e^y + x^3e^{2x}).$$

7. (§1.6: 31-42) Is the following equation exact? If so, solve it.

$$\left(x^3 + \frac{y}{x}\right) dx + \left(y^2 + \ln x\right) dy = 0.$$

8. (§1.6: 43-54) Reduce the following second-order equation to a first-order equation:

$$yy'' + (y')^2 = yy'.$$

9. (§2.1: 1-8) Solve the separable equation by using partial fractions

$$\frac{dx}{dt} = 7x(x - 13), \quad x(0) = 17.$$

10. (§2.1: 9-31) The time rate of change of an alligator population P in a swamp is proportional to the square of P. The swamp contained a dozen alligators in 1988, two dozen in 1998. When will there be four dozen alligators in the swamp? What happens thereafter?

11. (§2.2: 1-18) Find equilibrium solutions and their stability.

$$\frac{dx}{dt} = 3x - x^2.$$

12. (§2.3) Suppose that a motorboat is moving at 40 ft/s when its motor suddenly quits, and that 10 s later the boat has slowed to 20 ft/s. Assuming that the resistance it encounters while coasting is proportional to its velocity, how far will the boat coast in all?

13. (§2.4, but not computationally intensive) Using Euler's method with a step size of .1, estimate $y(.2)$ in the solution to

$$y' = -y, \quad y(0) = 1.$$