Review Sheet: Final (Solutions)

1.

$$a)\ y = \frac{1}{2}x^2 - \frac{1}{3}x + \frac{1}{2} + \frac{1}{12}x^{-2},$$

$$b)\ y = \sin 2x - x\cos^2 x + C\cos x.$$

2.

$$a)\ 0 < x < 3, \quad b)\ \frac{\pi}{2} < x < \frac{3\pi}{2}.$$

3.

$$a)\ \frac{1}{2}y^2 + e^y = \frac{1}{2}x^2 + e^{-x} + C,$$

$$b)\ y = \tan(\frac{1}{2}x^2 + x + C).$$

4.

$$a)\ \mathbb{R}^2, \quad b)\ x^2 + y^2 < 1.$$

5.

$$a)\ Q(t) = 100e^{\frac{1}{7}\ln(\frac{41}{50})t}, \quad b)\ t = -\frac{7\ln 2}{\ln\frac{41}{50}} \text{ days.}$$

6.

$$x^{'}(t) = \frac{3}{4} - \frac{3}{100}x(t), \quad x(0) = 10 \text{ lb.}$$

7.

$$t = 75\frac{\ln\frac{9}{13}}{\ln\frac{10}{13}} \text{ minutes.}$$

8.

$$a)\ x(t) = -\frac{gm}{c} - \frac{mA}{c}e^{-\frac{c}{m}t} + B, \quad v(t) = x^{'}(t).$$

9.

$$a)\ v = \pm\sqrt{v_0^2 - 2gR + \frac{2gR^2}{R+x}} \text{ where } R = \text{ radius of the earth,}$$

$$b)\ v_0 = \sqrt{2gR\frac{\xi}{R+\xi}}, \quad c)\ v_e = \sqrt{2gR}.$$

10.

$$a)\ x^2 + xy^2 = C, \quad b)\ x^3y + \frac{1}{2}x^2y^2 = C.$$

11.

$$a)\ y = \frac{Cx^2}{1-Cx}, \quad b)\ Cx^2 = (y-4x)^{\frac{3}{4}}(y+x)^{\frac{1}{4}}.$$

12.

$$a)\ \text{Linearly Independent}, \qquad b)\ \text{Linearly Dependent}.$$

13.

$$\text{No.}$$

14.

$$a)\ y = \frac{10}{13}e^x \cos 2x + \frac{2}{13}e^x \sin 2x, \qquad y = -\frac{1}{5}xe^{-x}.$$

15.

$$a)\ y = -3\sin x \cos 2x + \frac{3}{2}\ln|\csc x - \cot x|\sin 2x + 3\cos x \sin 2x + c_1 \cos 2x + c_2 \sin 2x,$$

$$b)\ y = \frac{1}{4}x(\ln x)^2 - \frac{1}{4}x\ln x + c_1 x + c_2 x^{-1}.$$

16.

$$a)\ x(t) = \frac{1}{6}\cos(8\sqrt{3}t) - \frac{1}{8\sqrt{3}}\sin(8\sqrt{3}t),$$

b) period $= \frac{2\pi}{\sqrt{192}}$ seconds, amplitude $= \sqrt{\frac{19}{576}}$ feet, phase $= -\tan^{-1}(\frac{\sqrt{3}}{4})$ radians.

17.

$$\omega = 8\sqrt{3}.$$

18.

$$a)\ \overrightarrow{x}(t) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t},$$

$$b)\ \overrightarrow{x}(t) = c_1 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{-t/2} + c_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} e^{-t/2},$$

$$c)\ \overrightarrow{x}(t) = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} \right].$$

19.

$$a)\ \Phi(t) = \begin{pmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{pmatrix},$$

$$b)\ \Phi(t) = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} e^{-t/2},$$

$$c)\ \Phi(t) = \begin{pmatrix} e^{2t} & te^{2t} \\ -e^{2t} & -(1+t)e^{2t} \end{pmatrix}.$$

In all of the above $e^{At} = \Phi(t)\Phi^{-1}(0)$.

20.

$$x_1(t) = 10 + 5e^{-.6t}, \qquad x_2(t) = 5 - 5e^{-.6t}.$$