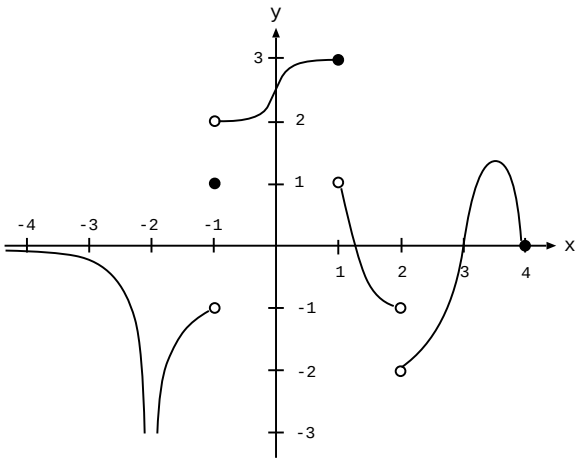
MAT 131 – CALCULUS I – FALL, 1999
MIDTERM EXAMINATION I

October 7, 1999

Name:	**ID Number:**	**Section:**

Problem	1	2	3	4	5	6	**Total**
Score							

Problem 1. (20 points) Below is the graph of $y = f(x)$. Answer each of the following questions.



(a) What is $f(-1)$?

(b) What is $f(2)$?

(c) What is $\lim_{x \to -2} f(x)$?

(d) What is $\lim_{x \to -1^+} f(x)$?

(e) What is $\lim_{x \to 1} f(x)$?

(f) What is $\lim_{x \to -\infty} f(x)$?

(g) What is $\lim_{x \to 4^-} \cos(f(x))$?

(h) Is f continuous at $x = 3$?

(i) Is f continuous at $x = 1$?

(j) What is $f(f(1))$?

Problem 2. (15 points) Using the graph of $y = x^2$, arrange the following numbers in ascending order (you do not have to compute the values):

- the slope s_a of the line tangent to the graph at $x = 1$;
- the slope s_b of the line tangent to the graph at $x = -1$;
- the slope s_c of the line tangent to the graph at $x = 2$;
- the slope s_d of the chord of the graph between $x = 1$ and $x = 2$ (*i.e.*, the line joining the points on the graph at $x = 1$ and $x = 2$);
- the number 0.

Problem 3. (20 points) Suppose that f is a *continuous* function with domain $[0, 5]$. Below is a table giving *some* of the values of f. You are given this limited information about f and nothing more. For each of the following statements, say whether it is true, false, or unknown. *Explain your answer.* (No credit is given without an explanation.)

x	$f(x)$
0	-5
2	10
3	0.1
4	0.01
5	0.001

Hint. It will help to graph f.

(a) $\lim_{x \to 2} f(x) = 10$.

(b) There is some number $a \in [0, 5]$ that $f(a) = 0$.

(c) The range of f is the interval $[0, 5]$.

(d) $f(2.5)$ lies between 0.1 and 10.

(e) f possesses an inverse.

Problem 4. (5 points) Let $f(x) = e^x$, $g(x) = x^2$, $h(x) = -x$, and $k(x) = e^{-x^2}$. Express k as a composition, in some order, of f, g, and h.

Problem 5. (15 points) Suppose that a train leaves from Stony Brook Station for Penn Station every hour. The train doors close precisely on the hour. So as long as you arrive on the platform any time before that, you will catch the train; but if you arrive exactly on the hour, you will have to wait an hour for the next train. Each train takes exactly two hours to reach Penn Station.

Let $f(x)$ be the elapsed time from your arrival on the platform until your arrival at Penn Station if you arrive on the Stony Brook platform x hours after midday. (For example, $x = 2.5$ means you arrive on the platform at exactly 2:30 pm.) Assume that you catch the first available train after arriving on the platform.

(a) Draw a graph of $f(x)$ for $x \in [0, 3]$. (Be sure to label the axes, and clearly indicate the values of $f(x)$ at $x = 0$, $x = 1$, $x = 2$, and $x = 3$.)

(b) What is $\lim_{x \to 1^+} f(x)$?

(c) What is $\lim_{x \to 2^-} f(x)$?

(d) What is $\lim_{x \to 1.5} f(x)$?

Problem 6. (25 points) Calculate the following limits, showing your work.

(a) $\lim_{x \to 3^-} \dfrac{x^2}{x-3}$

(b) $\lim_{x \to 1} \dfrac{x^2-1}{x-1}$

(c) $\lim_{x \to -2} \dfrac{x}{x+2}$

(d) $\lim_{x \to -\infty} \dfrac{3x^3+x}{x^3+2x+1}$

(e) $\lim_{x \to 1} f(x)$ given that $x - \frac{1}{2}x^2 \leq f(x) \leq 1 - x + \frac{1}{2}x^2$