MAT 131 – CALCULUS I – FALL, 1999
FINAL EXAMINATION

December 21, 1999

Name:	**ID Number:**	**Section:**

Problem 1. (20 points) Compute the following indefinite and definite integrals.

(a) $\displaystyle\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

(b) $\displaystyle\int \sec^2\theta \, d\theta$

(c) $\displaystyle\int \left(7t^2 - 14t + 1\right) dt$

(d) $\displaystyle\int \left(\frac{2}{x} - \frac{2}{x^2}\right) dx$

(e) $\displaystyle\int_1^2 \left(2 + t - 6t^2\right) dt$

(f) $\displaystyle\int_{-2}^{-1} 3e^x \, dx$

(g) $\displaystyle\int_0^{\pi} \sin\theta \, d\theta$

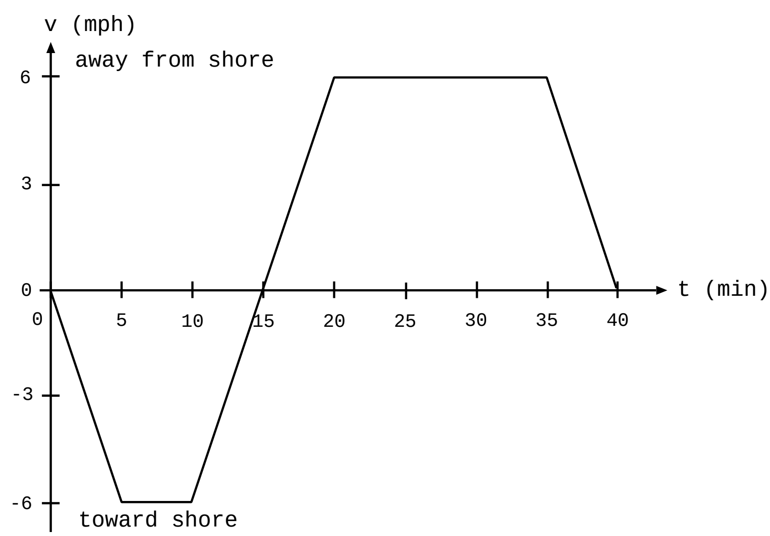
(h) $\displaystyle\int_0^1 \frac{1}{1+u^2} \, du$

Problem 2. (10 points) Let $f(x) = \sqrt{1-x^2}$.

(a) Find the equation of the line tangent to the graph of f at $x = 1/2$.

(b) When $a = .4$, $f(a) \approx .92$ and $f'(a) \approx -.44$. Use the tangent line approximation (*i.e.*, linear approximation) to approximate $f(.39)$.

Problem 3. (10 points) Lisa is enjoying rowing a row boat on a lake. Starting from a standstill when she is two miles from shore, she first rows toward shore and then turns around and rows away from shore. The velocity of the row boat in *miles per hour* is graphed as a function of time in *minutes* below.



(a) What is Lisa's distance from shore when $t = 40$ minutes?

(b) When (*i.e.*, at what t) is Lisa closest to shore?

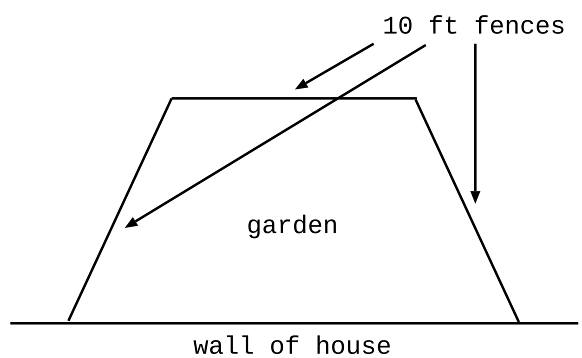
Problem 4. (10 points) The derivative of $x\ln(x)/(1+\cos x)$ is

$$\frac{(1+\cos x + x\sin x)\ln x + 1 + \cos x}{1 + 2\cos x + \cos^2 x}.$$

Calculate

$$\int_1^e \frac{(1+\cos x + x\sin x)\ln x + 1 + \cos x}{1 + 2\cos x + \cos^2 x}\,dx.$$

Problem 5. Mrs. Jones wants to build a fenced garden behind her house and asks you to help her determine the best shape. "I want it as large as possible," she says, "but I only have these three pieces of fencing and the wall of the house to work with." You measure the pieces and find that they are each 10 feet long. She also requests that the garden look symmetrical, by which she means that one piece of fence should be parallel to the house and the other two pieces should have the same angle against the house. (See the diagram below.)



(a) (10 points) Find a function of one variable whose maximum determines Mrs. Jones' ideal garden. Be sure to explain what is the variable you choose.

(b) (optional, for 10 points extra credit) Find the maximum value of this function and say what shape Mrs. Jones' ideal garden has.

Problem 6. (20 points) Let $f(x) = x^3 - 3x$.

(a) Find the local and global maxima and minima of $f(x)$ for $-5/2 \leq x \leq 3/2$.

(b) For which values of x between $-5/2$ and $3/2$ is $f(x)$ concave down?

Problem 7. (10 points) Calculate the following derivatives.

(a) $\dfrac{d}{dt}\left(5t^4 - 3t^3 + \dfrac{17}{2t} - \dfrac{t}{6} + 83\right)$

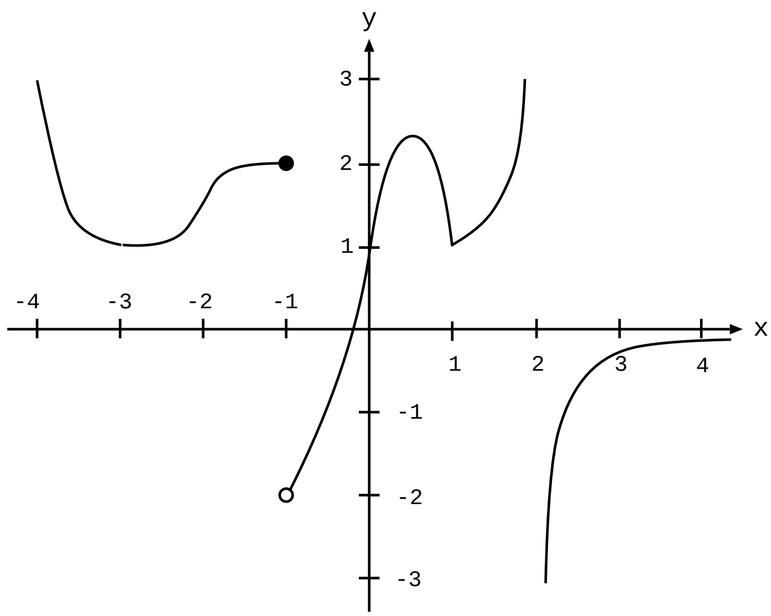
(b) $\dfrac{d}{d\theta}\left[(1+\theta)\cos^2\theta\right]$

(c) $\dfrac{d}{dx}\dfrac{1}{1+x^2}$

(d) $\dfrac{d}{dt}e^{-0.05t}$

(e) $\dfrac{d}{dr}\ln(1+r^2)$

Problem 8. (10 points) The graph $y = g(x)$ appears below.



(a) What is $\lim_{x\to -1^+} g(x)$?

(b) What is $\lim_{x\to 2} g(x)$?

(c) For what values of x does $g(x)$ fail to be differentiable?

(d) For what values of x between -4 and -1 does $g(x)$ have an inflection point?

(e) For what values of x between -1 and 1 is $g(x)$ concave down?

Problem	1	2	3	4	5	6	7	8	**Total**
Score									