MAT 362 SPRING 05 HOMEWORK 8

Due Thursday, April 20

1. Do Problem 7.19, p.169 in the text, and make sure you understand the steps.

For Problems 2 and 3, you might want to use Corollary 10.2, p.234 of the text.

2. Compute the Gauss curvature of a surface whose first fundamental form in a chart is

$$du^2 + f^2(u)dv^2,$$

where f is a function only of u. Does the answer you get remind you of anything you've already seen?

3. Suppose σ is a conformal chart for a surface, so that the first fundamental form in this chart has coefficients

$$E = G = \lambda, \; F = 0,$$

where λ is a function of u, v. Show that the Gauss curvature is given by

$$K = -\frac{1}{2\lambda}\Delta(\log \lambda),$$

where Δ is the Laplacian on $\mathbb{R}^2$: $\Delta\phi = (\partial^2\phi/\partial u^2) + (\partial^2\phi/\partial v^2)$.

4. Compute the Gauss curvature for an open set in the plane $\mathbb{R}^2$
(a). In Cartesian coordinates.
(b). In polar coordinates.
Verify that your answers are the same in both coordinates.

5. Show that no neighborhood of a point in a sphere can be mapped isometrically into a plane, i.e. there is no isometry from a neighborhood of a point on $S^2(1)$ mapping into the flat plane. .